Note on the Computation of Sample Size for Ratio Sampling

Valerie LeMay, Ph.D., RPF
Forest Resources Management
University of B.C.
Vancouver, BC, CANADA

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Background

Ratio sampling is commonly used to reduce confidence intervals for a variable of interest (**Y**), based on the information obtained for a related variable (**X**) in design dependent sampling[1]. Using the **X** information and ratio sampling, the estimated total for the **Y** variable is:

$$\hat{Y}_R = \hat{R} \times X \qquad [1]$$

where

$$X = \sum_{i=1}^{N} x_i$$

$$\text{and} \quad \hat{R} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i x_i} = \frac{\hat{Y}_{HT}}{\hat{X}_{HT}} \quad \text{and} \quad w_i = \frac{1}{\pi_i} \qquad [2]$$

The probability that item i appears in sample of *n* observations is given by π_i (called the inclusion probability), y_i and x_i are measures for the selected item, HT indicates the Horvitz--Thompson estimator; *N* is the number of items in the finite population; and *n* is

[1] In model dependent sampling, the model determines how variability is estimated, and the sampling design is not important as long as the model holds. In design dependent sampling, the sampling design determines how variability is estimated (Lohr, 1999). There is great debate on which approach is better for different situations. Schreuder et al. (1993) and Lohr (1999) present short discussions on this topic and refer the readers to other debates presented in literature. Lohr (1999) explains that the difference in variances is due to the definition of variance. In model dependent sampling, the variance is the average squared deviation of the estimator from the expected value, but this is averaged over all samples that could be generated from the population model (population is assumed to be infinite and follows the model given; Schreuder et al., 1993, termed this a superpopulation). In design dependent sampling, the variance is the also the average squared deviation of the estimator from the expected value, but this is averaged over all samples that could be obtained using a given design.

the number of items selected from the population. The ratio estimator results in a biased estimate of the total for **Y**. However, this bias decreases with increasing sample size, and is some cases will be zero (Schreuder, et al. 1993). If samples are taken with replacement, the approximate variances are:

$$V(\hat{Y}_R) = V(\hat{Y}_{HT}) + R^2 \times V(\hat{X}_{HT}) - 2 \times R \times COV(\hat{Y}_{HT}, \hat{X}_{HT}) \qquad [3]$$

and

$$V(\hat{R}) = V(\hat{Y}_R / X) = V(\hat{Y}_R) / X^2 \qquad [4]$$

If sampling is without replacement, an additional term must be added to the approximate variance given in [3], and this term is dependent upon the sampling design used.

Simple random sampling

For simple random sampling (SRS; sampling with equal probability), with or without replacement, the estimated total becomes:

$$\hat{Y}_R = \hat{R} \times X$$

where

$$X = \sum_{i=1}^{N} x_i \quad and \quad \hat{R} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i x_i} = \frac{\sum_{i=1}^{n} \frac{N}{n} y_i}{\sum_{i=1}^{n} \frac{N}{n} x_i} = \frac{y_T}{x_T} = \frac{\sum_{i=1}^{n} y_i \Big/ n}{\sum_{i=1}^{n} x_i \Big/ n} = \frac{\bar{y}}{\bar{x}} \qquad [5]$$

since $\pi_i = n/N$ for both with and without replacement. The approximate variances can be estimated by:

$$\hat{V}(\hat{Y}_R) = \left(1 - \frac{n}{N}\right) \times \left(\frac{N^2}{n}\right) \times \left(s_y^{\ 2} + \hat{R}^2 \times s_x^{\ 2} - 2 \times \hat{R} \times s_{xy}\right) \qquad [6]$$

and

$$\hat{V}(\hat{R}) = \hat{V}(\hat{Y}_R / X) = \hat{V}(\hat{Y}_R) / X^2 \qquad [7]$$

where $s_y^{\ 2}$, $s_x^{\ 2}$, and s_{xy} are the usual sample estimates of the variances of Y and X, and the covariance of X and Y. The finite population correct factor is removed if sampling is with replacement. Cochran (1977) gives alternative equations for the approximate variances for sampling with replacement as:

$$V(\hat{Y}_R) = \frac{N^2(1-n/N)}{n} \times \left[\frac{\sum_{i=1}^{N} (y_i - R \times x_i)^2}{N-1} \right]$$

$$V(\hat{\bar{Y}}_R) = V(\hat{Y}_R) / N^2$$

$$V(\hat{R}) = V(\hat{Y}_R) \ / \ (N^2 / \bar{X}^2)$$

which are equivalent.

The sample size for ratio estimation using simple random sampling is:

$$n = \frac{t^2 \times s_1{}^2}{AE^2} = \frac{t^2 CV_1{}^2}{PE^2} \quad where \quad s_1{}^2 = s_y{}^2 + \hat{R}^2 \times s_x{}^2 - 2 \times \hat{R} \times s_{xy} \qquad [8]$$

where AE is the allowable error for estimating the population mean. Instead of the allowable error, the percent error (allowable error expressed as a percent of the estimated mean, where the estimated mean is the total given in [1] divided by *N)*, and the CV (standard deviation expressed as a percent of the estimated mean) can be used. For sampling without replacement, the equations become:

$$n = \frac{1}{\frac{1}{N} + \frac{AE^2}{t^2 \times s_1{}^2}} = \frac{1}{\frac{1}{N} + \frac{PE^2}{t^2 CV_1{}^2}} \qquad [9]$$

Unequal probability sampling with replacement

For unequal probability sampling, equations [1] and [2] are used to estimate the total and ratio. For sampling with replacement, the inclusion probability for item *i* is:

$$\pi_i = 1 - (1 - \psi_i)^n = 1 - P(\text{unit } \ i \ \text{ is not in the sample})$$

where ψ_i is the probability that unit *i* is selected in the first draw (Lohr, 1999). Equation [2] can then be simplified to (Schreuder et al. 1983, page 46):

$$\hat{R} = \frac{\frac{1}{n}\sum_{i=1}^{n} y_i / \psi_i}{\frac{1}{n}\sum_{i=1}^{n} x_i / \psi_i} = \frac{\hat{Y}_{HT}}{\hat{X}_{HT}} \qquad [10]$$

Since $\pi_i \cong n \times \psi_i$ when sampling is with replacement. The following estimator has been proposed for the approximate variance of the total.

$$\hat{V}(\hat{Y}_R) = \hat{V}(\hat{Y}_{HT}) + \hat{R}^2 \times \hat{V}(\hat{X}_{HT}) - 2 \times \hat{R} \times C\hat{O}V(\hat{Y}_{HT}, \hat{X}_{HT}) \qquad [11]$$

where

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^{n} \frac{\left(y_i/\psi_i - \hat{Y}_{HT}\right)^2}{n(n-1)} \qquad [12]$$

and the equation is similar for $\hat{V}(\hat{X}_{HT})$. However, it is not clear what the estimator of $C\hat{O}V(\hat{Y}_{HT}, \hat{X}_{HT})$ should be (Schreuder et al. 1993). Schreuder *et al.* (1993, pages 94 and 95) give five variance estimators, based on work by Sarndal (1980 and 1982). For sampling with replacement, the first two variance estimators result in zero values (Equations 3.148 and 3.149 of Schreuder et al.). The three remaining estimators are:

$$\hat{V}_3(\hat{Y}_R) = \sum_{i=1}^{n} \frac{(1-\pi_i)e_i^2}{\pi_i^2} = \sum_{i=1}^{n} (1-\pi_i)(w_i e_i)^2 \ \ where\ \ e_i = y_i - \hat{R} \times x_i \qquad [13]$$

$$\hat{V}_4(\hat{Y}_R) = \sum_{i=1}^{n} \frac{(1-\pi_i)e_i'^2}{\pi_i^2} \ \ where\ e_i'\ is\ a\ \mathrm{mod}ification\ of\ e_i\ (see\ Schreuder\ et\ al) \ [14]$$

$$\hat{V}_5(\hat{Y}_R) = \left[\sum_{i=1}^{n} \frac{e_i^2}{\pi_i^2} + \frac{1}{2} \times \sum_{i \neq j}^{n} \frac{e_i \times e_j}{\pi_{ij}}\right] \times \frac{(n-1)}{n} = \left[\sum_{i=1}^{n} \frac{e_i^2}{\pi_i^2} + \frac{1}{2} \times \sum_{i \neq j}^{n} \frac{e_i \times e_j}{\pi_i \pi_j}\right] \times \frac{(n-1)}{n} \qquad [15]$$

The third estimator is also given in Sarndal (1984 and 1994) for the specific case of the ratio of means estimator with the variance proportional to the **X** variable. Lohr (1999, page 357) give another variance estimator as:

$$\hat{V}_L(\hat{Y}_R) = \frac{\hat{V}\left(\sum_{i=1}^{n} w_i q_i\right)}{\left[\sum_{i=1}^{n} w_i x_i^2 - \left(\sum_{i=1}^{n} w_i x_i\right)^2 \Big/ \sum_{i=1}^{n} w_i\right]^2} \qquad [16]$$

$$where\ q_i = (y_i - \hat{R} \times x_i)(x_i - \bar{x}_w)\ \ and\ \ \bar{x}_w = \hat{X}_{HT} \big/ \hat{N} = \hat{X}_{HT} \Big/ \sum_{i=1}^{n} w_i$$

Lohr also shows how this variance estimates differs from the model based estimator used by statistical packages, even when the sampling is based on SRS. All of these estimators are based on the sum of the weighted errors, or on a transformation of the weighted errors.

The variance of the ratio can then be found using:

$$\hat{V}(\hat{R}) = \hat{V}(\hat{Y}_R / X) = \hat{V}(\hat{Y}_R) / X^2 \cong \hat{V}(\hat{Y}_R) / (\hat{N} \times \bar{x}_w)^2 = \frac{\hat{V}(\hat{Y}_R)}{\bar{x}_w^{\,2} \times \left(\sum_{i=1}^{n} w_i\right)^2}$$

Instead of using a function to estimate the variance of an unequal probability ratio of means sample, many authors suggest the use of a Taylor series, or resampling methods such as Balanced Repeated Replication (BRR) and Jackknifing.

Because there is no clear selection of the variance estimator for unequal probability sampling, calculating an approximate sample size is difficult. Alternatives for calculating the sample size are as follows:

1. The design effect is calculated as the ratio of the variance of the unequal probability design, relative to the variance of a simple random sample of the same size. If the design effect in using unequal probability sampling instead of equal probability sampling was known or could be approximated, then equation [8] using simple random sampling would be multiplied by the estimated design effect. This is difficult to apply unless knowledge of the design effect has been obtained through many surveys or through simulations that are applicable to the survey being conducted. Kish and Frankel (1974) stated that in general design effects for complex statistics are greater than 1. Therefore, using the sample size calculation for SRS ratio sampling would likely result in an underestimate of the expected number of observations.

2. Another alternative is to use the Equation [12] to obtain an estimate of sample size based on unequal probability sampling with replacement for Y only as:

$$n = t^2 \times \sum_{i=1}^{n} \frac{\left(y_i/\psi_i - \hat{Y}_{HT}\right)^2}{(n-1)} \Big/ AE^2 \qquad [17]$$

   without the additional information on the X variable, where the AE is the allowable error for the total of Y. This would result in an overestimate of the expected sample size, since this would be a less efficient design. Kish and Frankel (1974) suggest that this results in "safe overestimates". If the allowable error is expressed for the mean instead of the total, the equation becomes:

$$n = t^2 \times \sum_{i=1}^{n} \frac{\left(y_i/\psi_i - \hat{Y}_{HT}\right)^2}{(n-1)} \Big/ (AE^2 \times N^2) \cong t^2 \times \sum_{i=1}^{n} \frac{\left(y_i/\psi_i - \hat{Y}_{HT}\right)^2}{(n-1)} \Bigg/ \left(AE^2 \times \left(\sum_{i=1}^{n} w_i\right)^2 \right)$$

   If the allowable error is given for the ratio, this must be converted to an allowable error for the total by:

$$AE(total) = AE(ratio) \times X \cong \quad AE(ratio) \times \bar{x}_w \times \hat{N} = AE(ratio) \times \bar{x}_w \times \sum_{i=1}^{n} w_i$$

Equation [17] can then be used.

3. A third alternative is to define the design effect as the ratio of the variance of the estimated total using unequal probability ratio of means sampling over the variance of the total for unequal probability sampling. The resulting design effect would then be multiplied by the sample size calculated for unequal probability sampling. Using the equations given above, this would be:

$$n = \frac{\hat{V}(\hat{Y}_R)\left(Eq.13,14,15,16\, or\, resampling\, estimate\right)}{\hat{V}(\hat{Y}_{HT})\left(Eq.12\right)} \quad \times \quad n\ (Eq.[17]) \qquad [18]$$

<u>Out of these three options, the third alternative appears most feasible since a reasonable estimate of the sample size needed could be obtained using previously collected data for the area. However, since there are two estimates of the variances involved in the calculation, the sample size would likely vary greatly for different sets of previously collected data.</u>

Unequal probability sampling without replacement

If sampling is without replacement, the calculations of the inclusion probabilities are more difficult, since the probability of selecting item i in a given draw of a sample unit depends on what happened in previous draws. Also, estimated variances are more difficult to calculate. As an alternative, the variance estimators for sampling with replacement could be used. These would result in an overestimate of the variances, since sampling with replacement is less efficient than sampling without replacement (Lohr, 1999). A similar approach could be used for calculating the sample size.

Weighted least squares regression versus least squares regression using sampling weights

A common approach to incorporating sampling weights into a linear model (including ratio of means sampling) is to use weighted least squares regression in standard statistical packages. If weights are calculated as the inverse of the inclusion probabilities as in equation [2], then statistical packages that allow the use of weights for regression will result in the same estimate of the ratio (slope of the regression) as equations [1] and [2]. However, the variance estimates for weighted least squares package are only appropriate if the weights are equal to the inverse of the variance of the Y values (Holt *et al.* 1980; Schreuder *et al.* 1993; Lohr 1999). The reason for this is that the weighted least squares given in standard regression packages results in the following estimates of the coefficients and of the variances:

$$\hat{\beta} = (X^T W X)^{-1} X^T W y$$
$$Var(\hat{\beta}) = \sigma^2 (X^T W X)^{-1}$$

where the weights are on the diagonal of the square W matrix, and the ratio is the estimated slope. This is appropriate when the weights are the inverse of the variances. If sampling weights are not proportional to the inverse of the variances, the coefficients are estimated in the same manner, but the variance of those coefficients is:

$$Var(\hat{\beta}) = \sigma^2 (X^T W X)^{-1} X^T W W X)(X^T W X)^{-1}$$

(Korn and Graubard 1995; Lohr 1999, page 361). For ratio sampling, both the sampling weights and the variance weights need to be incorporated into the weight matrix.

Example for Unequal Probability Sampling

Sample data for 105 trees were supplied by Sam Otukol of the Ministry of Forests, Resources Inventory Branch. The data included the actual felled volume (Y; ANMV) and the cruiser called net volume (X; CNMV). The list of data and all calculations are included as Appendix I.

The data were initially graphed. As expected, the ANMV is very similar to the CNMV. CNMV is found by calculating the gross merchantable volume using a taper function by Kozak, and then subtracting the percent of decay estimated by the cruiser in the field. Since the taper function is very precise, and the decay percent is commonly low, the CNMV values are very similar to the ANMV values. Based on the graph, the sample size needed to obtain an estimate of the total volume for the population is expected to be much lower if the CNMV information is used along with the felled tree information.

Two different values for the allowable error were used to illustrate the recommended approach for calculating sample size (item 3 under **Unequal Probability Sampling with Replacement**). Although samples are selected without replacement, the approach for sampling with replacement is recommended and used here, since 1) this method is much easier to calculate and 2) this results in an overestimate of the sample size needed (conservative estimate), as noted in **Unequal Probability Sampling without Replacement**. A 95% probability (t or z approximately equal to 2) was used.

The results were that $\hat{V}(\hat{Y}_{HT}) = 3447494.45$, if the CNMV information is not used, and $\hat{V}(\hat{Y}_R) = 91043.31$, if the CNMV information is used. The estimated total was 11,127 in both cases. Setting the allowable error for the total as 15% of the estimated total (AE = 1669), the sample size without the CNMV information was calculated as n=520, and with the CNMV information was n=14 (14 felled trees are needed). Setting the allowable error for the ratio as 10% (ratio within 0.09 units), the allowable error for the total was

1113. The sample size without the CNMV information was n=1169, and with the CNMV information was n=31 (31 felled trees are needed).

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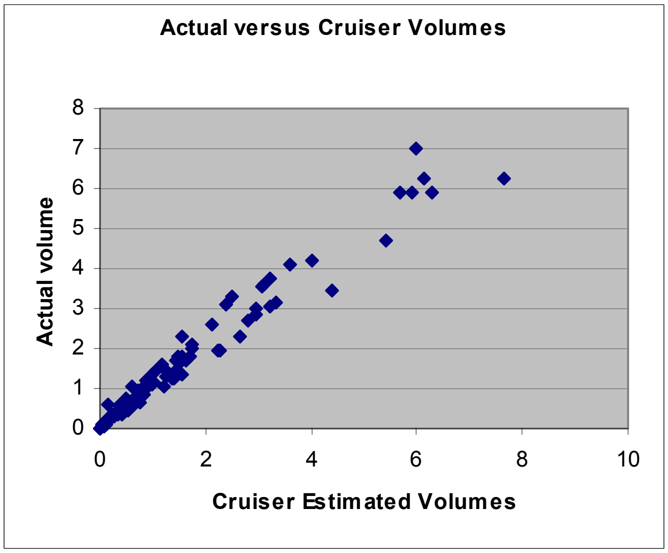
Appendix I. Example for Calculation of Approximate Sample Size Using Ratio Sampling with Unequal Probability of Selection



Nhat:	Xhat:		Yhat:				
6332.107397	11772.33942		11126.60929				
Rhat:	n:		V(Yhat [12]):		V(Yhat [13]):		Deff [13] / [12]
0.945148529	105		3447494.455		91043.3104		0.026408544
	Examples	for sample	size	estimation	using 95%	probability:	
			1)	AE=15%	of Yhat:	1669	
				n [Eq. 17]:		n [Eq. 18]:	
				519.8046896		13.72728523	
			2)	AE=10% of	Rat:	0.094514853	
				convert to	AE for Yhat:	1112.660929	
				n [Eq. 17]:		n [Eq. 18]:	
				1168.860111		30.86789415	

CNMV (X)	wi	xi*wi	ANMV (Y)	yi*wi	phi	pi	yi/phi
1.969227462	133.0396675	261.9853669	2.225699773	296.1063578	7.15862E-05	0.007516555	31091.16757
0.465156723	241.6547846	112.4073478	0.543720436	131.3926448	3.94108E-05	0.004138134	13796.2277
2.709637348	123.3616621	334.265367	2.807421011	346.3281223	7.72023E-05	0.008106246	36364.45284
3.065563318	178.3654422	546.7905568	3.213998197	573.2662096	5.33949E-05	0.005606467	60192.95201
0.357194079	57.47463492	20.52959928	0.234544797	13.48037656	0.000165705	0.01739898	1415.439538
0.01965176	6.197640671	0.121794545	0	0	0.001536683	0.161351723	0
0.028680941	4.409293168	0.126462677	0.023353304	0.102971563	0.00215994	0.22679372	10.81201414
0.362749374	51.01751461	18.50657147	0.263071051	13.42123118	0.000186677	0.019601112	1409.229274
0.440060499	24.43157161	10.75136958	0.343844402	8.400659134	0.000389816	0.040930646	882.0692091
1.678114657	31.19389458	52.34693168	1.609957913	50.22085741	0.00030531	0.032057555	5273.190028
1.267211103	45.05964511	57.10008259	1.397130357	62.95419808	0.00021136	0.022192807	6610.190798
6.250410141	57.500884	359.4041085	7.63848111	439.2194163	0.000165629	0.017391037	46118.03871
1.816291293	36.49112938	66.27852058	1.544075628	56.34506354	0.00026099	0.02740392	5916.231671
0.322519443	32.55931909	10.50101344	0.231025482	7.5220324	0.000292506	0.030713173	789.813402
0.873828229	56.80758048	49.64006745	0.718369457	40.80883073	0.00016765	0.017603284	4284.927227
0.694536212	22.44488405	15.58878475	0.682157811	15.31095296	0.00042432	0.044553583	1607.650061
1.152501574	72.99459921	84.1263905	1.02281372	74.65987755	0.000130473	0.013699644	7839.287143
0.142923669	34.36914014	4.9121636	0.11322739	3.891528022	0.000277104	0.029095869	408.6104423
0.840110424	19.15178372	16.08961314	0.747618297	14.31822393	0.000497281	0.052214458	1503.413512
3.100772509	21.10996562	65.45720106	2.382555956	50.2956743	0.000451152	0.047370991	5281.045802
1.340804361	77.09523939	103.3696332	0.989823679	76.31069347	0.000123533	0.01297097	8012.622815
0.794515261	22.79407145	18.11023764	0.695649635	15.85668749	0.00041782	0.043871057	1664.952187
0.338711644	41.31816415	13.9949433	0.326741485	13.50035833	0.000230499	0.024202431	1417.537624
0.035692237	4.108012604	0.14662416	0.093315622	0.383341751	0.00231835	0.243426712	40.25088387
0.591997088	15.68714429	9.286743738	0.394650855	6.190944906	0.000607109	0.063746465	650.0492152
0.092209177	8.007449659	0.738360343	0.095611938	0.765607778	0.001189369	0.124883707	80.38881667
0.580206888	12.33810067	7.158650991	0.140915942	1.73863508	0.000771902	0.081049752	182.5566834
0.731857558	56.9753885	41.69786871	0.496873996	28.30958894	0.000167157	0.017551438	2972.506839
3.010290902	13.34183584	40.16280703	2.937430412	39.19071434	0.000713831	0.074952204	4115.025006
3.578904187	34.48013397	123.4010958	3.124755378	107.7419841	0.000276211	0.029002207	11312.90833
1.449860726	13.80565189	20.01627248	1.047747363	14.46483536	0.000689849	0.072434102	1518.807713
1.814197364	17.88183159	32.44117173	1.48318782	26.52211482	0.000532597	0.055922683	2784.822056
4.116270519	40.82541984	168.0484721	3.610493893	147.399929	0.000233281	0.024494543	15476.99255
1.275364986	104.4600353	133.2246714	1.234372477	128.9425925	9.11718E-05	0.009573039	13538.97221
1.052934655	277.6389108	292.3356309	0.833438024	231.3948252	3.43029E-05	0.003601801	24296.45665
0.337959708	65.23955418	22.04834067	0.435440748	28.40796029	0.000145982	0.015328124	2982.83583
3.296547827	238.439515	786.0272653	2.517405632	600.248978	3.99422E-05	0.004193936	63026.14269
0.053232707	7.720026143	0.410957893	0.030399401	0.234684174	0.00123365	0.12953324	24.64183823
0.02519655	3.648944648	0.091940817	0.025234136	0.092077966	0.002610018	0.274051841	9.66818643
0.689320718	22.87721412	15.76973765	0.6814301	15.5892223	0.000416301	0.043711616	1636.868341
0.152058014	7.140553487	1.085778385	0.060921706	0.435014699	0.001333763	0.140045166	45.6765434
0.02443451	70.55787487	1.724047103	0	0	0.000134979	0.014172762	0
0.274878981	10.36442375	2.848962238	0.182295351	1.889386268	0.000918894	0.096483898	198.3855581
0.278841528	9.449946048	2.635037391	0.250844365	2.370465715	0.001007816	0.10582071	248.8989001
0.520387125	13.43580372	6.991819275	0.390312257	5.244158873	0.000708838	0.074428	550.6366816
0.092035703	5.148590703	0.473854164	0.031739412	0.163413244	0.00184979	0.194227908	17.15839058
0.25584548	28.79160222	7.366201287	0.164730746	4.742862112	0.000330784	0.03473235	498.0005218
1.178382526	11.94575946	14.07667421	0.869289231	10.38432006	0.000797254	0.083711714	1090.353606
2.117676296	22.61537225	47.89203773	1.728467895	39.08994486	0.000421121	0.044217711	4104.44421
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0.420406679	121.2830081	50.98818664	0.394924315	47.89760892	7.85255E-05	0.008245178	5029.248936
0.597923932	88.09899841	52.67649954	0.627944092	55.32124559	0.000108103	0.011350867	5808.730786
0.63241261	87.91967068	55.6015084	0.749328588	65.88072268	0.000108324	0.011374019	6917.475881

yi/phi-Yhat	col.I sq.	ei	wi*ei	(wi*ei)^2	(1-pi)*col.M
19964.55857	398583598.9	0.364487391	48.49128129	2351.404361	2333.729901
2669.618703	7126864.018	0.104078256	25.15100862	632.5732346	629.9555615
25237.84384	636948761.5	0.246411336	30.39771203	924.0208964	916.5305555
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-11115.79699	123560942.6	-0.003754444	-0.016554446	0.00027405	0.000211897
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-10244.53979	104950595.5	-0.072078118	-1.760981701	3.101056552	2.974128306
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