Nearest Neighbor Methods for Imputing Missing Data Within and Across Scales

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Mapping/Assessment Problem

Measures for all variables of interest and for all scales of interest are not available

Example:

- Forested land, divided into polygons (stands, same age, species, etc.) – complete census based on photos/remote sensing
- Ground data are available for some of the stands
- Wish to "populate" the forested land with detailed information

Imputing Missing Data

Imputation involves estimating missing values for variables of interest

Many methods and variations:

- Univariate (one variable of interest at a time) vs multivariate (all variables of interest simultaneously)
- Single values or means from existing data as estimates for missing values
- Requires probability distribution or can be distribution-free
- Spatial information or variable-space?

Univariate Methods

- Sample means used to impute missing values

e.g all trees with missing heights get average height of 30 m (98 ft), regardless of their diameter

- Generate a random value from a sample estimated distribution
- Use regression or logistic models

E.g. diameter = 50 cm (20 in), predicted height= 30 m (98 ft) Trees of dbh=50 cm without measured heights assigned an estimated height of 30 m.

Issues with Univariate Methods

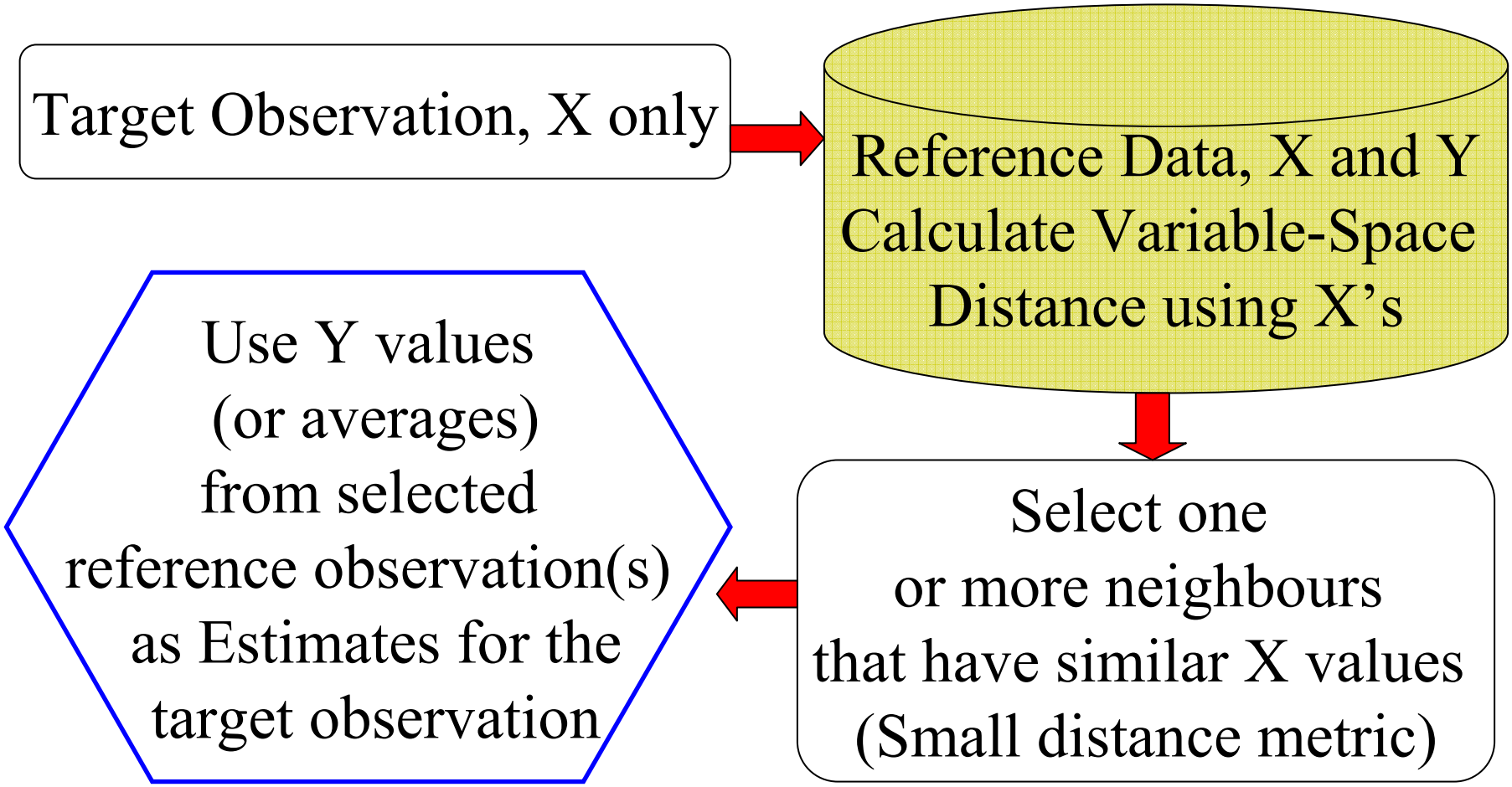
- For means and regression, variables must be ratio or interval scale
- All are unbiased and statistically consistent estimates (if models are correct)
- Only random selection from a probability distribution retains variability (means lowest)
- No assurance of logical consistency across several variables of interest

Multivariate Nearest Neighbor Imputation Methods

Data

- Obtain a sample on which X's (auxilary variables) and Y's (variables of interest) are measured [reference data set]
- Can have many Y's
- X's and Y's can be class and/or continuous variables (will affect the methods used)
- On all other observations of the population, measure the X's only [target data set]

Imputation Steps in General



Imputation: Example

For:

Use:

Distance (Similarity) Metrics

- A number of possible metrics
- Distance in variable-space
- Different measures if some are class variables

Squared Euclidean Distance

$$d_{ij}^{\,2} = (X_i - X_j)'(X_i - X_j)$$

X_i = vector of standardized values of the

X variables for the *i*th target observation

X_j = a vector of standardized values of the

X variables for the *j*th reference observation

Most Similar Neighbor Distance = Weighted Euclidean Distance

$$d_{ij}^{\,2} = (X_i - X_j)'W(X_i - X_j)$$

W = weight based on canonical correlation between X and

Y variables using the reference data

Other Distance (Similarity) Measures

- City Block
- Manhattan

} For Class Variables

- Absolute Difference

Variations

Single or Weighting of Many Reference Observations:

- Select one substitute? Or average more than one? Weighted or unweighted average?
- Affects degree of "smoothing" of estimates

Pre-stratification or not?

- E.g., by ecozone? By region?

(Single) Nearest Neighbor (NN)

- Select the closest reference observation (smallest distance)
- Values for all Y variables from the nearest neighbor are the estimates for the target observation
- E.g., Moeur and Stage used NN with their distance metric, Most Similar Neighbour

Tabular Nearest Neighbor

- Stratify reference data into groups
- Calculate variable averages (tables) by group
- Calculate similarity for X variables between a target observation and table averages
- Select the closest table
- Use the table average values for the Y's as the estimates for the target observation

k-Nearest Neighbors (k-NN) and Weighted k-NN

- Select the *k* most similar observations from the reference data
- Average the values for all Y variables from the *k*- nearest neighbors; averages are the estimates for the target observation
- For weighted k-NN, calculate a weighted average of the k-neighbors (e.g., 1/distance as the weight); weighted averages are the estimates for the target observation

Properties: Not Necessarily Unbiased

Over all samples, the mean bias (bias = average difference between observed and estimated value) does not necessarily equal zero for Y or X variables

- For Y: match is based on X variables, not Y
- For X: match may have lowest distance, but not the lowest difference, and compromised among variables

Properties: Bias Example

Target: X1=2 X2=4

Reference 1: X1=0 X2=4 Y1=10 Y2=5

Reference 2: X1=1 X2=3 Y1=7 Y2=4

Ref. 1 better for X2 (squared Euclidean distance of 4)

Ref. 2 better for X1 (squared Euclidean distance of 2)

Properties: Not Necessarily Statistically Consistent

- The average distance between target and match observations tends to decline with increasing sample size (**more likely to find a close match**)
- But mean bias will not necessarily decline with increasing sample size
- Why? Variables that are "hard to find a match for" influence the distance more

  e.g. X1=300 X2=10 Will try to find a match for the extreme X1 value and sacrifice X2.

Properties: May Retain Variability

- Retains the variability of the variables over the population if a single neighbor is used to impute missing values of a target observation
- If many neighbors are selected (k-NN) variation is not retained
  - similar to regression and other models, except that this is multivariate

Properties: Logical Consistency

- Logical consistency across several variables if using one neighbor
  - the combination of variables must exist in the population
- Using averages of many nearest neighbors: some logical inconsistencies may arise

  e.g., volume by species – Ref. 1 has pine and aspen and Ref. 2 (next closest) has larch and spruce. Average will have all four species

Other Properties

- Computationally Intensive: Need similarity between the target observation and each of the reference observations
- Generally, better correlations between the X's and the Y's yield better imputation results
- Multivariate Estimation: can obtain estimates of all the Y variables simultaneously
- Variables of interest can be class or continuous variables or mixed
- Distribution-free

Selecting a Nearest Neighbor: Demonstrations of Issues

Photo 1

Q. 1
Want Coarse Woody Debris and Snags for **Photo 2**

Photo 3

Photo?
X-Variables?

Photo 2

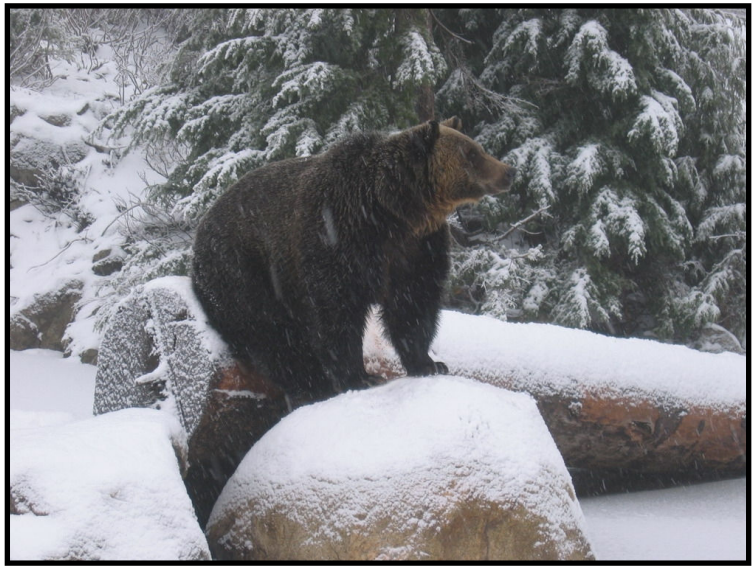


Photo 4 (Yikes!)

Observations

- May be very difficult to obtain the reference data you need
- X-variables matter

Photo 1

Q. 2

Photo 3

Want soil moisture/ nitrogen for **Photo 3** Photo?
X-Variables?

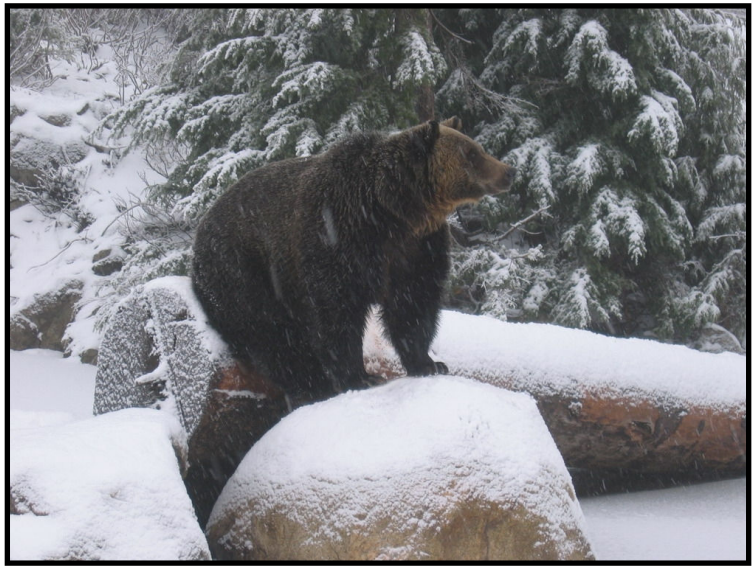


Photo 2

Photo 4

Observations

- Stratifying by location should be considered
- For some variables, time of year when measures are taken are important

Research into Forestry Applications

Examples and Results of Testing Using Simulations

- Tree-lists: X-stand level; Y-tree level
- Regeneration: X-overstory; Y-understory, both at stand-level
- Other Applications:
  - Volume and basal area per ha: X-aerial variables; Y-ground variables both at stand-level (Forest Science Paper)
  - Wildlife Trees: X-stand level; Y-tree level (Conference Proceedings)

Estimating Tree-Lists

- A tree-list (stems per ha by species and diameter) for every polygon would be useful
  - for projecting future stand volume, and
  - for estimating current and future stand structure, as inputs to habitat models
- Can we obtain reasonable estimates of tree lists for non-sampled polygons, based on aerial information?

Data

- 96 polygons were ground-sampled using variable radius plots (Y)
- Up to 9 species in a polygon with a wide diameter range
- Aerial variables (X) were matched to the ground data

Variable Set

Y variables (7):

- basal area/ha
- stems/ha of Douglas fir(D), larch (L), and lodgepole pine (PL)
- Max. dbh of F, L, and PL

X variables (8)

- Percent crown closure
- Average height (m)
- Average age (yrs)
- Site index (m)
- Percents of F, L, and PL by crown closure
- Model estimated volume/ha (stand level model)

Methods:

- SAS 6.12 used to simulate sampling the population (100 replicates)
- Three sampling intensities (20%, 50% and 80%)
- Two imputation methods used: Tabular and Most Similar Nearest Neighbor (NN with MSN Distance)

Correlations Between Ground and Aerial Variables

- Highest for stems per ha of fir (Y) with model estimated volume per ha (X) (about 0.40)
- Lowest for Maximum dbh of larch (Y) with crown closure class (X) (less than 0.01)

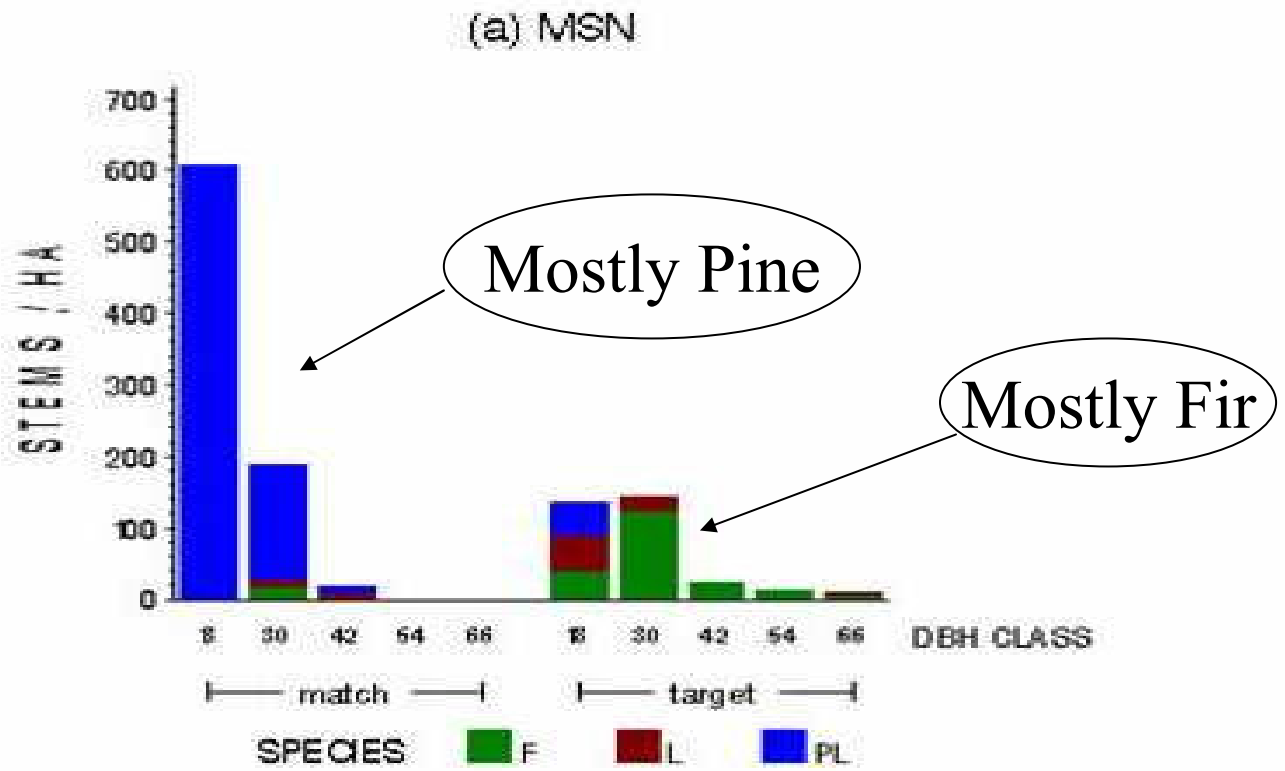
Results Over 100 Replications

- Average correlations between targets measured and imputed variables:
  - For X: Increased as sample size increased
  - For Y: Generally increased with sample size but not for all variables (e.g., decreased for stems/ha larch using MSN)

Results Over 100 Replications

- Mean Bias (average difference) for Y:
  - Generally lower for Tabular than MSN
  - Not declining with increasing sample size
- Mean of Mean Squared Errors for Y:
  - Declined with increasing sample size for most variables
  - MSN and Tabular similar

Example of Target and Match Polygons (80% Sampling Intensity)



Estimating Regeneration Under an Overstory After Partial Cutting

- Stands are multi-species and multi-aged, partially cut; measure overstory variables (X)
- Want to estimate the amount of regeneration (Y) expected to occur following partial cutting
  - Regeneration by 4 species groups by 4 height classes and all very related
- Tabular and MSN (NN with Most Similar Neighbor Distance)

Tabular Imputation: E.g., Dense, Dry (n=18), <6 years after cutting (stems/ha)

Species	Height (cm)				Total
	15-49.9	50-99.9	100-129.2	>130	
Tolerant	3921	1032	454	495	5903
Semi-tol.	2889	949	372	578	4788
Intolerant	1197	41	41	0	1280
Hardwood	454	248	248	743	1692
Total	8462	2270	1115	1816	13663

Imputation Accuracy Over Cells

Match: Presence of regeneration in both the target

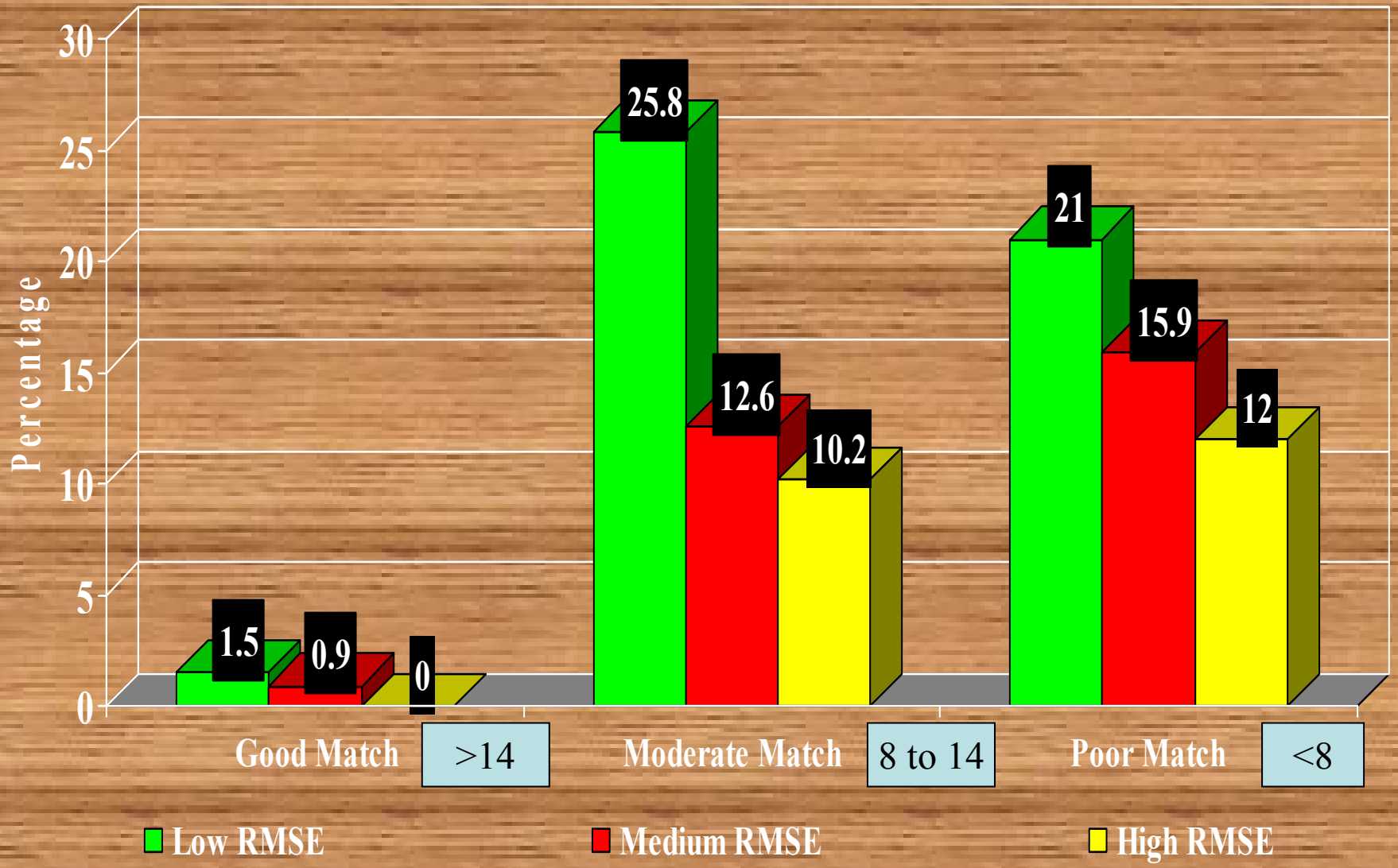
Good (>14 cells matched)	moderate (>8 to 14)	poor (<8)

Grouped plots also by root mean squared error

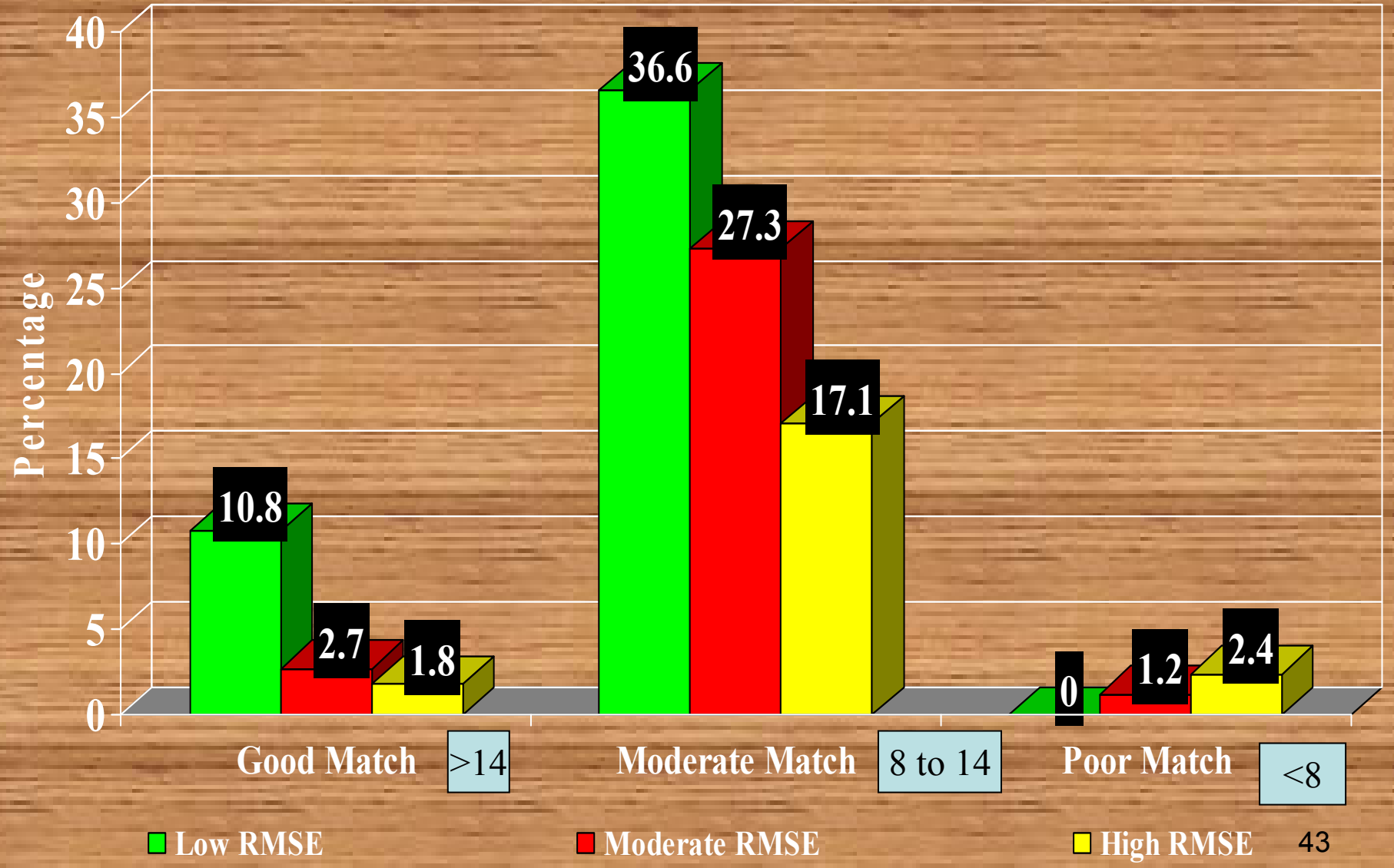
low (<1000 stems per ha, all species)	
moderate (1000-2000)	high (>2000)

Want Good, Low

Performance of Tabular



Performance of MSN



Comparison of Approaches

- Better estimates using MSN
  - MSN uses a single nearest neighbor – variability and logical consistency retained
  - Tabular can be considered “smoothing” (k-NN also is smoothing) – for this problem, too much “smoothing” likely

Summary for Imputation Methods

- Imputation methods are used to fill in missing data for variables of interest across and within scales
  - Can be used to “fill in” data needed for long term monitoring, such as within stand details needed for risk mapping
- Many methods and variations on methods

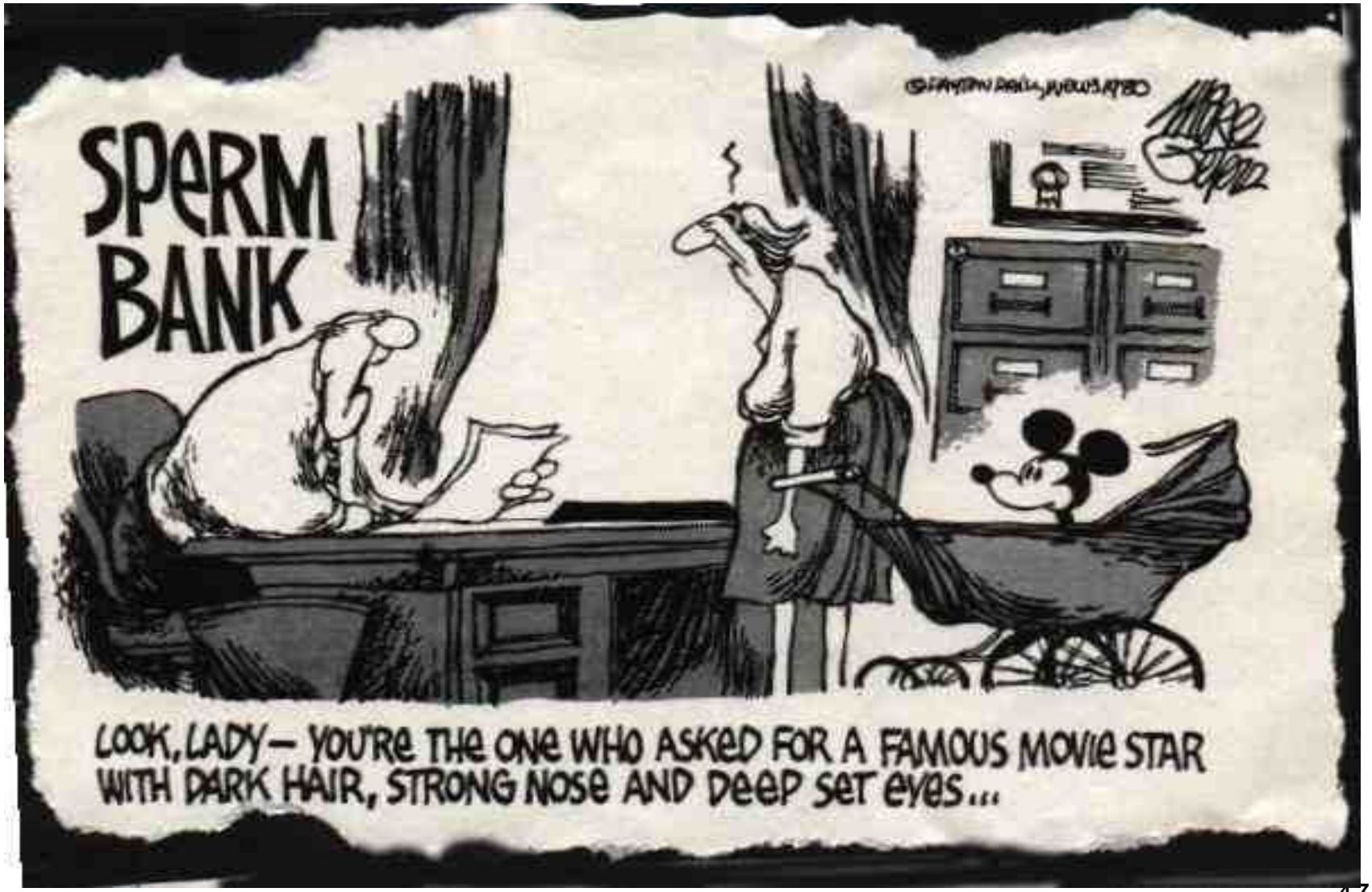
Summary for Imputation Methods

Nearest neighbor methods

- are multivariate and distribution-free
- can retain logical consistency and variation
- can be used for class or continuous or mixed variables of interest

- Degree of "smoothing" – from single nearest neighbor to k-NN to Tabular – can adversely affect accuracy of results
- Need a "good" set of reference data, with auxiliary variables that are well related to variables of interest

X-variables matter



Websites and Acknowledgements

Articles:

www.forestry.ubc.ca/Prognosis

www.forestry.ubc.ca/biometrics

NN Software (website given on the Abstract also):

forest.moscowfsl.wsu.edu/gems/msn.html

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