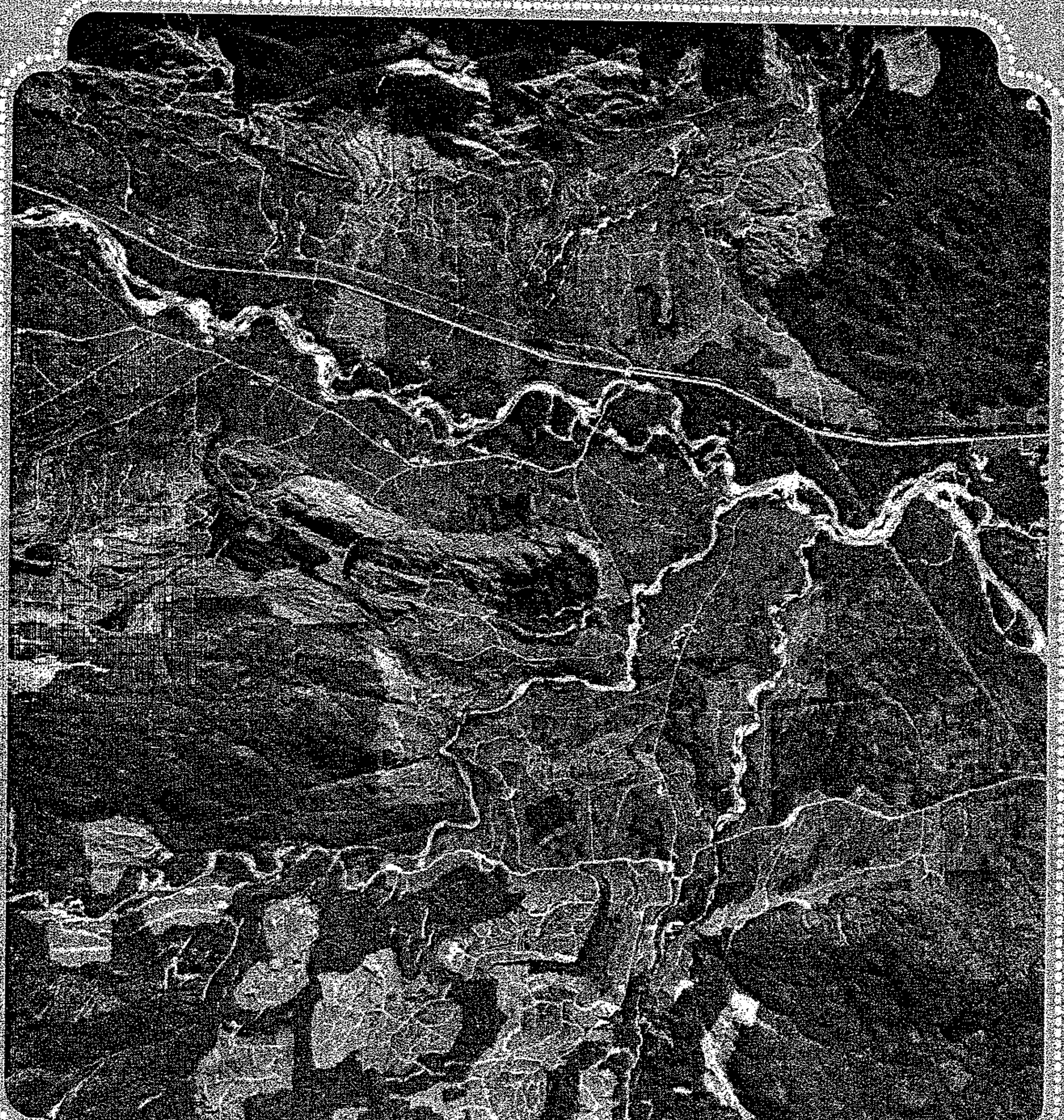
Forestry 237

Introduction to Forest Mensuration and Photogrammetry



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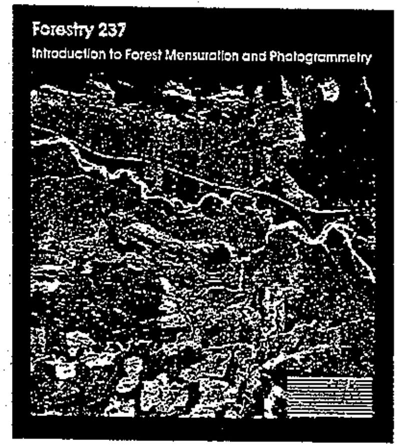

FORESTRY 237:

INTRODUCTION TO FOREST MENSURATION AND PHOTOGRAMMETRY

FRST 237 COURSE MANUAL

P. L. Marshall

V. M. LeMay

Distance Education & Technology
Continuing Studies
The University of British Columbia
Vancouver, B.C., Canada

FRST 237

Course Manual Authors

Dr. Valerie M. LeMay is a faculty member in the Department of Forest Resources Management, Faculty of Forestry at the University of British Columbia.

Dr. Peter L. Marshall is a faculty member in the Department of Forest Resources Management, Faculty of Forestry at the University of British Columbia.

Course Administration

Beth Hawkes
Program Director, Distance Education and Technology, Continuing Studies

Production Credits

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FRST 237

INFORMATION ABOUT THE COURSE

COURSE DESCRIPTION

FRST 237 (3 credits) *'Introduction to Forest Mensuration and Photogrammetry'* - Measuring and estimating tree volumes, form, and taper. Timber scaling and grading. Computer applications. Basic photogrammetry, mapping from photography, and photo-based inventory systems.

Prerequisite: Forestry 231, 232 (or equivalents).

INTENDED STUDENT

The prospective student must have had a course in introductory statistics covering the rudiments of probability theory, confidence interval construction, simple linear regression, and one-way analysis of variance (i.e., Forestry 231 or equivalent). An introductory course in computing and/or familiarity with spread sheet or statistical software on a microcomputer would be a definite asset (e.g., Forestry 232 or equivalent). Some explanations of the mensurational techniques require an understanding of differentiation and integration. Access to a microcomputer would be beneficial for some of the assignments, but is not essential.

This course is designed for:
- students who, prior to entering the Faculty of Forestry at UBC, want to complete some of the required courses in the B.S.F. program, through a combination of college courses and/or off-campus independent study courses;
- pupils within the ABCPF program who want to complete the course requirements in mensuration. This course constitutes the prerequisite for FRST 238, which together with FRST 237, meets these requirements;
- any person who wants to learn more about forest mensuration and photogrammetry.

COURSE CONTENT

This course is divided into eight lessons. The first four lessons provide an introduction to mensuration; the latter four provide an introduction to photogrammetry and photo interpretation.

1. *Basic Mensurational Measurements* - an overview of what mensuration entails, and the theory and procedures for measuring tree diameter at breast height, tree height, and upper stem diameters;
2. *Measurement of Tree Volume* - principles relating to the measurement of tree volume, including total stem volume, merchantable volume, and net volume;
3. *Estimation of Tree Volume* - methods of estimating tree volume from measurement of other related attributes, covering form factors, form quotients, volume equations, and taper equations;
4. *Log Scaling* - approaches to determining the volume and quality of logs;
5. *Introduction to Photogrammetry and Photo Interpretation* - introduction to photogrammetry and photo-interpretation, including the concepts of vertical aerial photography, characteristics of light, and the photographic process;
6. *Principles of Photogrammetry* - the basic geometric theory behind the techniques for measuring the heights of objects on vertical aerial photographs;

7. *Mapping Systems and Mapping from Aerial Photography* - map projection techniques and mapping systems, and planimetric mapping from aerial photographs, including theory behind planimetric mapping, ground control points, photo control points, flight planning, radial line triangulation, and topographic mapping from aerial photographs;
8. *Applications of Photogrammetry and Photo Interpretation in Forestry* - forestry applications of non-photographic imaging systems, photogrammetry and photo-interpretation, including forest cover typing, species identification, photo volume equations, and applications in forest inventories.

Course Objectives

Upon completion of this course, you will have:

1. gained a good understanding of the rationale, basic theory, instruments, and techniques for measuring certain tree attributes from the ground and from aerial photographs;
2. learned some tree measurement techniques that can be used any where in the world with some emphasis on techniques presently used in British Columbia;
3. become familiar with the theory behind planimetric and topographic map construction from aerial photographs.

Course Materials

Materials for this course consist of this course manual, required textbooks, recommended supplementary readings, and a laboratory kit of equipment.

course manual

This manual contains lesson commentaries, instructions regarding the course readings and directions for the completion and submission of assignments. The lesson commentaries serve to supplement the textbooks, and also to emphasize important issues and concepts.

required textbooks

One textbook is required for the course. It can be purchased from the UBC Bookstore, using the order form provided.

Avery, T.E. & Burkhart, H.E. 1994. ***Forest Measurements.*** 4th ed. New York: McGraw-Hill.

supplementary reading
Extension Library list

The following books are recommended reading for this course; they can be obtained from the Extension Library, and some items, as indicated, can be purchased from the UBC Bookstore or directly from the Ministry of Forests.

Avery, T.E. & Berlin, G.L. 1985. ***Interpretation of Aerial Photographs.*** 4th ed. Minneapolis: Burgess Publishing. [Out of print; the Extension Library may be able to obtain a copy of the book from other library branches.]

HUSCH, B., MILLER, C.I. & BEERS, T.W. 1982. ***Forest Mensuration.*** 3rd ed. New York: John Wiley. [Out of print.]

MINISTRY OF FORESTS. 1980. ***Scaling Manual.*** Province of British Columbia, Victoria. [because of subscription update service, order from Ministry]

PAINE, D.P. 1981. ***Aerial Photography and Image Interpretation for Resource Management.*** New York: John Wiley. [Available from Bookstore by special order.]

WATTS, S.W. (Editor). 1983. ***Forestry Handbook for British Columbia.*** 4th ed. Vancouver: University of British Columbia, Faculty of Forestry, Forestry Undergraduate Society. [Available as a stocked item from Bookstore]

laboratory kit

The kit contains the following: Suunto clinometer; dbh tape; 30 m tape; stereoscope; and height finder.

This equipment is necessary in order to complete certain of the assignments. Borrowing procedures and deposit requirements are outlined in a cover letter sent to you with the kit.

HOW TO PROCEED THROUGH THE COURSE

As indicated earlier, the course is made up of eight lessons divided into two parts: Lessons 1 through 4, and Lessons 5 through 8. These two parts essentially stand alone, but we recommend that you proceed through the lessons in the order given. Within either part, it is important that you fully understand each lesson before moving on to the next. You should first read the lesson in the manual and try any practice calculations given in the commentary. Next, you should read relevant sections in the text books. These are given in the introduction to each lesson. Before you attempt graded assignments (only six of the eight lessons have graded assignments), you should answer the relevant review/self-study questions. These question sets, at the end of each lesson, are also intended to provide a guide to studying the material prior to the final exam.

COURSE REQUIREMENTS

Course requirements include six assignments which you must submit for grading and comments, a two-day laboratory session to be held at UBC, and a final examination. In addition, each lesson has a number of self-study questions and activities which provide you with an opportunity to test your understanding of the material and develop skills.

graded assignments

There are six assignments, one for each of Lessons 1, 3, 4, 5, 6, and 7. Directions on how to complete each assignment are provided on the assignment sheets in Appendix A. If any other supplemental material not covered in the lesson is required to complete the assignment, this is provided as well. The assignments should be completed and submitted to your tutor for grading by the dates indicated on your course schedule. All assignments must be completed before you can participate in the laboratory session.

laboratory session

A two-day laboratory session will be held at UBC after you have completed the material in the course manual. The lab session must be completed satisfactorily before you can write the final exam. A portion of the lab session will consist of field work. During this portion, you will have an opportunity to practice using a number of mensurational instruments too bulky or too expensive to send to you in your lab kit. The remainder of the session will consist of office work. Most of this time will be spent in a computer lab where you will be instructed in how to use a microcomputer statistical package. You will also have an opportunity to practice some elementary statistical analyses on data that you collected during the field portion of the session. A report covering the various activities you participated in will be due at the end of the session.

final examination

There will be a three-hour final examination at the end of the course. Questions on any portion of the course may be included on this examination. Material examined will be divided equally between the two portions of the course.

Grading

The grade for this course will be broken down into the following percentages:

Assignments	30%
Laboratory Session	20%
Final Examination	50%

In order to pass this course, you must:

1. complete and submit all assignments before attending the laboratory session;
2. attend the lab session and complete the report prior to writing the final exam;
3. obtain a passing mark (≥ 50%) on the final exam;
4. obtain an overall passing mark (≥ 50%) for the course.

Role of the Tutor and Tutor/Student Communication

Upon registration, you will be assigned a course tutor. The tutor will be available for telephone communication during specific hours each week. You can call your tutor at those times free of charge. Please see the instructions on the free telephone service contained in the *Student Handbook*. The tutor can also phone you if you complete and submit the blue student telephone form in the *Student Handbook*.

In addition, you can send brief questions to the tutor by mail when you submit your assignments. The tutor will also include written comments on each assignment when it is returned.

Student Evaluation of the Course

At the end of this course, a course evaluation form will be sent to you for completion. This is an opportunity for you to express your opinion on course content and presentation, administrative procedures, and delivery method. Your responses will be of great value in improving the course for future students.

LESSON 1 BASIC MENSURATIONAL MEASUREMENTS

INTRODUCTION

LESSON OVERVIEW

The purpose of this lesson is to provide an overview of what mensuration entails, and to cover the theory and procedures for measuring tree diameter at breast height, tree height, and upper stem diameters. The associated assignment will provide practice using some of the measurement instruments described and a review of some basic statistical techniques. This lesson provides the background necessary for Lessons 2 through 4.

LESSON OBJECTIVES

After studying this lesson and completing the first assignment, you should be able:

1. to list and categorize factors involved in mensuration;
2. to measure tree diameter at breast height using a diameter tape;
3. to outline the theory behind two approaches to measuring tree height;
4. to use a staff hypsometer and a Suunto clinometer to measure tree height;
5. to calculate upper stem diameters from relascope measurements;
6. to determine horizontal and slope distances using relascope measurements.

LESSON READINGS

Material relevant to this lesson may be found in Avery and Burkhart pages 1–9 and 97–108.

LESSON ASSIGNMENT

When you have completed this lesson, be sure to complete both the self-study question set (at the end of the lesson) and the first graded assignment (in Appendix A). Mail the graded assignment to your tutor by the date indicated on the course schedule. Don't forget to include a pink assignment sheet.

FOREST MENSURATION

Forest mensuration has traditionally been associated with the measurement of tree characteristics and the products cut from them. Measurement of the volume of trees or stands of trees was of primary importance. As forestry has expanded and management of the forest has become more important, many measurements besides volume are required and many things other than trees are being measured in the forest. Whether or not the measurement of these other things should be considered as part of mensuration or part of a specific discipline (e.g., hydrology, wildlife management, etc.) depends upon how broadly mensuration is defined.

As an entity, forest mensuration deals with:

1. identifying the variables that are important for decision making in forest management (i.e., determining **what** to measure);
2. designing a sampling scheme to efficiently collect the information required at an appropriate level of precision (i.e., determining **where** and **how much** to measure);
3. measuring or estimating variables (i.e., determining **how** to measure);
4. analysing the data to provide the information desired (i.e., determining **what** the data contains).

Lessons 1 through 4 of this course address the measurement and estimation of certain characteristics of single trees (category 3). The UBC course Forestry 238 'Forest Mensuration' addresses category 2 and portions of categories 1 and 4.

PRECISION, BIAS AND ACCURACY

You frequently see these terms used in everyday conversation, often interchangeably. In mensuration, these terms each have a unique statistical interpretation.

precision

Precision refers to the spread of **observations** around the long-run mean of the observations. Observations are either individual measurements or statistics which combine many measurements. If individual measurements are used, precision is measured by the **standard deviation.** If statistics (e.g., the means of observations) are used, precision is measured by the **standard error** of the statistic. A precise measurement technique produces very similar results if the same items are measured by a number of different individuals.

bias

Bias refers to the difference between the long-run average of the observations and the true value. An unbiased measurement technique is one which yields the correct answer on average. Bias can be introduced by a faulty measurement instrument. As an example, consider a standard 25 cm ruler that is really only 24 cm long. Everything that you would measure with that ruler would systematically provide a measurement that was too large.

accuracy

Accuracy refers to how close an observation, or series of observations, are to being correct. It combines the elements of both bias and precision into a single measure of "goodness" and is often measured by the **mean square error.** An accurate measurement technique is precise and contains little or no bias.

A convenient way for you to keep these concepts clear is to visualize a series of targets (refer to Figure 1.1). The bull's eyes represent the true average value and the X's represent individual observations.

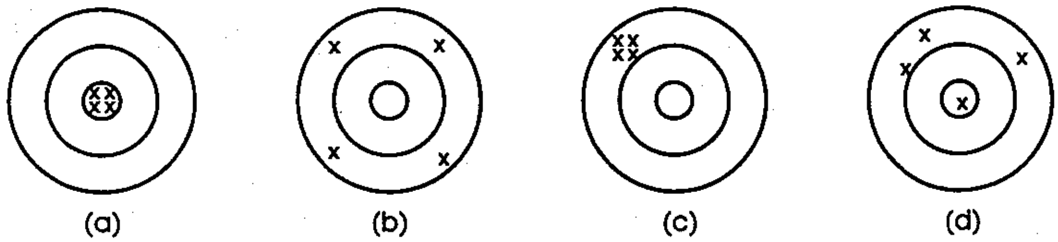


FIGURE 1.1 Relationship between precision, bias, and accuracy. **(a)** is both precise and unbiased; therefore, it is also accurate. **(b)** is unbiased, but not precise; therefore, it is not accurate. **(c)** is precise, but biased; therefore, it is not accurate. **(d)** is neither precise nor unbiased; therefore, it is not accurate.

SOURCES OF ERRORS

Errors in measurement or estimation can arise from a number of sources. We have classified the major sources below. All are controllable to some degree, but the sources may be difficult to quantify.

Instrument error

Instrument error arises from incorrect calibration of your measurement device. If it occurs, it is usually due to instrument damage or a calibration mistake. Examples include nylon chains that are stretched and compasses set with incorrect declinations. Instrument error usually contributes some bias to your measurements. These can sometimes be corrected after the fact, provided you are aware of the extent of the bias.

rounding error

Rounding error occurs when numbers are rounded. By their nature, instruments are only accurate up to a certain level of resolution. Attempting to go beyond that level will increase the variability of the results. For example, if a tape is graduated only in metres (m), measurements made using that tape and rounded to the nearest metre will obviously be more uniform than measurements taken to the nearest centimetre (cm). However, the results will not necessarily be as accurate since error is introduced by the rounding.

relationship error

Relationship error arises through an assumed relationship between variables. As an example, consider a regression equation of the form $Y = b_0 + b_1X$. You may be able to measure X exactly, but there will still be some error associated with the predicted Y value.

sampling error

Sampling error is a result of making inferences about a population based on a sample. This error source can be measured and controlled to some extent through the choice of certain sampling methods and regulating sample size. (This is discussed in some detail in Forestry 238.)

human error

Human error occurs as a result of mistakes. This type of error cannot be readily predicted nor recognized. The best way to minimize human error is through proper training and some sort of checking system.

TREE MEASUREMENTS

The question, "How do you measure a tree?" is difficult to answer because it is much too vague. There are many "things" that you can measure on a tree. These "things" are commonly called **attributes** or **characteristics.** For example, you can measure tree weight, tree height, bark thickness, and so on. The list is as long as the number of attributes a tree has. Furthermore, you must be specific as to what you want to measure. Even tree weight is not specific enough because the portion of the tree is not specified. Are roots to be included, or only the portion of the tree growing above ground? Do you mean to include branches and foliage? Is the bark to be included?

Attributes generally vary from tree to tree. For this reason they are often called **variables.** In a statistical sense, every tree may be described by a number of variables, many of which are not important for a particular task. In order to differentiate the important attributes from all the other possible attributes you could measure, the attributes that are considered to be important are often called **variables of interest.** Of course, whether or not a variable is important to you depends upon the purpose of your measurements.

In the remainder of this lesson, we will concentrate on the measurement of three tree variables that are frequently important: diameter at breast height, total tree height, and upper stem diameter. In Lessons 2 and 3, we will introduce the measurement and estimation of several types of tree volume. We will address scaling (the process of quantifying the woody material in logs) in Lesson 4.

MEASUREMENT OF DIAMETER AT BREAST HEIGHT

definition

Diameter measurements can be made at any point along a tree, branch, or log. These measurements can either include the bark, termed **diameter outside bark,** or exclude the bark, termed **diameter inside bark.** Outside bark diameters are by far the most common. If diameter inside bark is not explicitly stated or apparent from the context, you can assume that we are referring to diameter outside bark in this course. Diameters are commonly recorded in centimetres or millimetres (mm). The collective name for instruments that measure diameter is **dendrometer.**

The most common diameter measurement made on standing trees is taken at breast height and includes the bark. This measurement is referred to as **diameter at breast height** and commonly abbreviated as dbh. **Breast height** is defined as 1.3 m above some point close to the ground level (usually the estimated point of germination). Standardizing the location of diameter measurements on standing trees was necessary to allow for meaningful comparisons among measurements. Breast height is a convenient location because it can be easily reached by the individual making the measurement, yet is high enough on most temperate tree species to be above any butt swell that may be present. This latter point is important because the major use of dbh measurements is for estimating other variables that are less easy to measure (e.g., volume of wood in the main bole of the tree). Measurements of dbh also provide the **basal area** of a tree. The basal area of a tree is defined as the cross-sectional area of the tree at breast height and is calculated as ($\pi \times \text{dbh}^2 \div 4$). This is the area of a circle given a diameter equal to dbh. If dbh is measured in centimetres, then the basal area units would be in square centimetres (cm^2). To convert to square metres (m^2), simply divide the basal area by 100^2 or 10,000.

Other diameters are occasionally measured on a standing tree. For example, diameter at stump height (usually defined as 0.3 m above the point of germination) is sometimes measured in addition to dbh to provide a more accurate indication of the volume in the lower bole of the tree. Diameter at stump height, and any other diameters that can be reached by the individual making the measurement, are measured using the same instruments used to measure dbh. Diameters beyond the reach of the individual making the measurement (generally termed **upper stem diameters**) are also sometimes required. Instruments and procedures for measuring upper stem diameters differ from those used for measuring dbh. We begin by describing instruments for measuring diameters that can be reached; measurement of upper stem diameters is covered later in this lesson.

USE OF THE DIAMETER TAPE

You have been issued a diameter tape as part of your field kit. If you look at it while you read our description, it will make it easier for you to follow. The tapes are usually metal and contained in a closed spool which is metal or hard plastic. When the tape is on the spool it is difficult to damage, but it is possible to break the tape when it is extended if you pull it too hard. There is a pop-out handle on one side of the spool for winding in the tape. The tape itself has a hook at the free end for attaching to the bark. There are two scales on the tape. The scale that would be on the outside when the tape is hooked into the bark is calibrated in millimetres times π. This is the scale you will use for determining diameters. The scale on the flip side of the tape is in millimetres.

The fact that the scale used for determining diameters is in millimetres times π allows you to measure circumference, but record diameter. Circumference of a circle is equal to diameter times π, so if the diameter units are π times larger than standard, conversion occurs automatically. If the tree is not circular, then the diameter recorded will be positively biased. However, this is not usually of concern because the bias is quite small unless the tree is very far from circular.

To use the diameter tape, stick the hook into the bark at the appropriate height (e.g., 1.3 m above the point of germination to measure dbh). Then extend the tape around the tree in a plane that is perpendicular to the central axis of the tree. This means that if the tree is growing vertically on flat ground, the tape is extended around the tree parallel to the ground. If the tree is leaning and the ground is flat, the tape, in order to lie in the plane perpendicular to the central axis of the tree, will no longer be perpendicular to the ground (Figure 1.2). A common mistake when using a diameter tape is to allow the tape to sag when you extend it around the tree. This will positively bias your diameter measurements. The appropriate diameter is read off the tape as the point that coincides with the zero mark when it is wrapped completely around the tree.

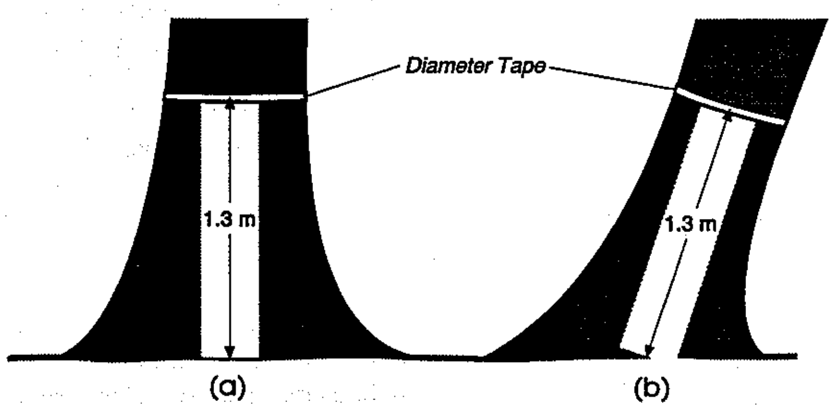


FIGURE 1.2 Proper positioning of the diameter tape on (a) a vertical tree and (b) a leaning tree.

Diameter tapes are commonly used for measuring dbh in North America. The major reason for this is the size of the instrument. Even a diameter tape large enough to measure the diameter of the largest trees found on the west coast is easy to carry and use. Another reason is that it is a consistent measurement device if used properly. In other words, if two people make the same measurement, they will get close to the same results.

Use of Calipers

Calipers consist of a calibrated flat shaft, a fixed arm at one end, and a movable arm (Figure 1.3). Calipers are made of either wood or steel, steel being more common these days. Because of their construction, calipers are practically indestructible.

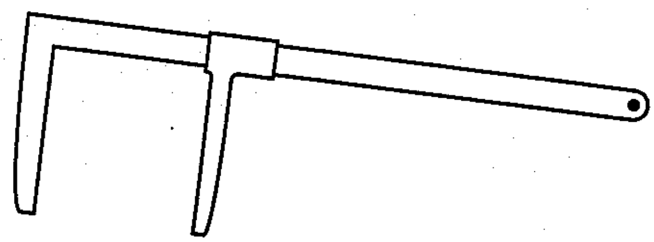


Figure 1.3 Calipers for measuring tree diameters.

Unlike a diameter tape, calipers measure diameter directly. The fixed end is placed against one side of the tree and the movable arm is slid tight against the opposite side of the tree. The diameter of the tree is read off the calibrated shaft. This requires the calipers to be longer than the tree is wide.

Calipers are better for determining diameters of noncircular trees than diameter tapes. On these trees, diameters can be measured in two directions perpendicular to one another and the results averaged. A quadratic average rather than an arithmetic (simple) average should be used if the diameter measurements are to be used for predicting tree volume since there is a better relationship. The quadratic average is calculated as:

$$\sqrt{\frac{d_1^2 + d_2^2}{2}}$$

where d_1 and d_2 are the two diameters.

Another advantage that calipers have over diameter tapes is speed. Diameters can be measured much more quickly using calipers. However, results using calipers may not be as consistent as those obtained using a diameter tape because the reading you take is dependent upon the direction in which you happen to approach the tree if the tree is not round. This is important only if very precise measurements are required. A more important disadvantage relative to the diameter tape is the size and weight of calipers. If diameters of large trees need to be measured, the calipers would need to be very large. A set of calipers is not included in your kit because their size makes it impractical.

Biltmore Stick

The **Biltmore stick** consists of a straight, graduated rule that is held horizontally against the tree. The zero end of the scale is placed so that it lies on the line of sight tangent to one side of the tree. The diameter is read off the scale by shifting

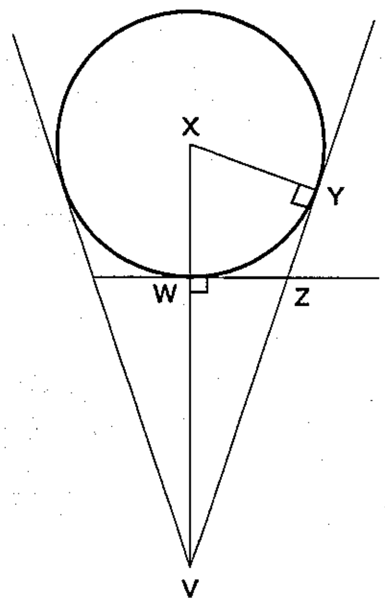


FIGURE 1.4 Illustration of the geometry underlying the calibration of a Biltmore stick.

the same eye to form a line of sight with the other side of the tree. In using the Biltmore stick, it is important that your eye be on the same level as the stick and at the distance from the stick for which the stick is calibrated. Biltmore sticks provide only approximate values for diameter, and so should be used only when rough answers are appropriate (e.g., for reconnaissance work).

You can build a Biltmore stick for yourself from a narrow board (about 1.5 m by 2 cm by 0.5 cm) and calibrate it for your own arm length. The calibration formula is:

$$B = \frac{D}{\sqrt{1 + \frac{D}{E}}}$$

where B is the distance from the zero point on the stick to the position for a given diameter D, and E is the perpendicular distance from your eye to the stick (approximately the length of your arm). This formula is derived by combining the theory of similar triangles with the Pythagorian Theorem using the basic diagram given in Figure 1.4. The following relationships will help in the derivation: (1) XW = XY = $D/2$; (2) EZ = $B/2$; (3) VW = E; (4) triangle VYX is similar to triangle VWZ.

MEASUREMENT OF TREE HEIGHT

TYPES OF HEIGHT

Total tree height is the distance between the ground and the tip of the tree. This is commonly referred to as **tree height** or simply **height.** Height measurements are commonly recorded in metres. Tree height is frequently measured because it is closely related to the volume in the bole of a tree and with other variables that are frequently of interest but difficult to measure (see Lesson 3). The heights of certain trees in a stand are also measured to help in estimating **site index** which is one means of quantifying site quality (i.e., the ability of a piece of land to produce timber of a certain species). Site index is described in detail in the UBC course Forestry 238.

The **height to the living or dead crown** is measured occasionally. This is the distance between the ground and the living or dead branches which comprise the crown. Height to the living or dead crown can be measured using the same instruments as those used for the measurement of total tree height.

Merchantable height also is measured occasionally. Merchantable height refers to the distance between the ground and some upper stem diameter that is considered to be the lower limit of merchantability (see Lesson 2). Instruments for measuring merchantable height require some means of determining diameters as well as height. One such instrument is the **relascope** which is discussed later in this lesson.

DIRECT MEASUREMENTS OF TREE HEIGHT

These measurements require that the measuring device be placed along the side of the tree. For this reason, direct measurements of height can be obtained easily only from felled or small trees. If the tree is felled, its height can be determined by measuring along the bole with a measuring tape. If the tree is small enough (generally under 5 m in height), its height can be measured using a **height pole.** Height poles consist of telescoping sections which can be extended along the side of the standing tree. When the pole is extended to the same height as the tree, the tree height can be read from a dial on the side of the pole.

Since we are often interested in obtaining heights for standing trees that are too tall to be measured with a height pole, **indirect** measurement techniques are frequently required.

Indirect Methods of Determining Tree Height

Indirect methods of determining tree height require the measurement of something other than tree height. The quantity that is measured is then used to calculate tree height. Techniques for indirectly measuring tree height are based on either trigonometry or geometry. Each of these procedures, along with descriptions of some of the instruments that are used with them, are discussed later in this section. Prior to this, we will cover the measurement of horizontal distances because this measurement is necessary for both approaches.

measurement of horizontal distances

If the ground is flat, **horizontal distance** (HD) can be measured along the ground surface using a **tape** or **chain.** Chains are named after the device originally used by surveyors for measuring horizontal distances. The original chains were composed of metal links and were chains in the technical sense. There was also a unit of distance measurement called a "chain" that equaled 66 feet (approximately 20 m). Metal tapes 132 feet long (2 "chains") replaced the original link chains, and were used in forestry for many years. Following the introduction of the metric system in Canada, these chains were slowly replaced by metric chains, commonly 50 m in length. Modern chains are usually made of nylon.

If the ground slopes, then either the tape can be held horizontally (this may sometimes require "breaking chain"), or the distance along the ground (slope distance) can be measured and corrected to give horizontal distance. If the ground is very steep, the latter technique is better.

In order to convert slope distance (SD) to horizontal distance, the angle of the slope must be known. The angle can be measured in either degrees or percent using a number of instruments. Possibly the most common of these is the Suunto clinometer.

Suunto clinometer

A **Suunto clinometer** is included as part of your kit. It is roughly rectangular in shape, approximately 7 cm long, 5 cm wide, and 1.5 cm thick (see Figure 1.5). The casing is metallic with three circular glass windows. A freely turning weighted wheel is visible through the largest window. The other two windows are on opposite ends. The largest of these is about 1 cm in diameter. This is the window that you look through (viewing window). The final window is not much larger than a pinhole. This provides some light for you to see the numbers on the scale. There is a metal ring on the end containing the viewing window. This ring should be on the bottom when you are looking through the instrument.

You will see a fixed horizontal line and a rotating wheel when you look through the viewing window. The line indicates where you should make the reading.

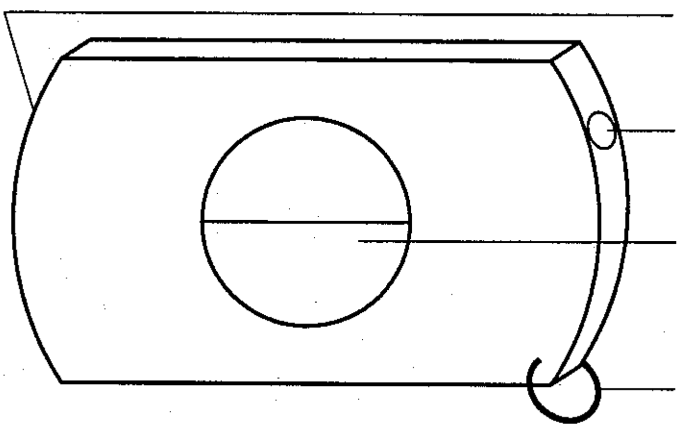


FIGURE 1.5 The Suunto clinometer.

There are two scales on the wheel. One is a percentage scale; the other is likely a degree scale. You can tell which is which by tilting the clinometer all the way up or all the way down while looking through the window. The scales are labelled at the ends of the graduated portions. The slope angle may be measured by looking at an object along the slope which is the same height above ground as your eye. It doesn't matter whether the slope is uphill or downhill when converting SD to HD. Only the absolute value of the angle is important.

If the slope angle (α) is recorded in degrees, a straightforward trigonometric correction can be used (see Figure 1.6).

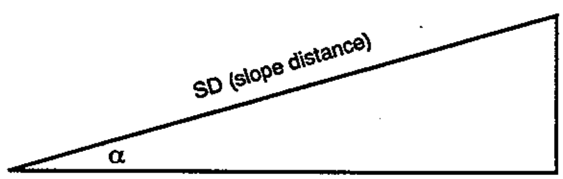


FIGURE 1.6 Relationship of slope distance to horizontal distance.

$$HD = SD \times \cos(\alpha)$$

As an example, consider a slope angle of 10 degrees and a slope distance of 20 m. The horizontal distance is:

$$HD = 20 \times \cos(10°) = 20 \times 0.985 = 19.7 \text{ m}$$

If the slope angle is recorded as a percentage, the percentage must be converted into degrees before the correction can be applied. The conversion is:

$$\text{degrees} = \arctan(\% \text{ angle} \div 100)$$

Arctan is the inverse of the tangent of the number. For example, the degree equivalent for a slope of 10% is:

$$\arctan(10 \div 100) = 5.71$$

For a slope distance of 20 m, the corresponding horizontal distance is:

$$HD = 20 \times \cos(5.71) = 19.90 \text{ m}$$

Sometimes slope corrections are added on to the end of the tape as a *trailer.* A trailer works best if the entire length of the tape is being used at one time (e.g., 50 m).

geometric approach

The principle underlying the **geometric approach** is that of similar triangles. This is a simple, approximate procedure to use when you want to find the height of a tree and do not have an angle measuring device with you. The instrument that is used is called a **staff hypsometer.** This is simply a straight pole that is somewhat longer than your arm, and weighted sufficiently at the bottom end to pull the staff vertical when it is held loosely in your hand at a distance equal to the length of your arm. In practice, any broken branch that meets the criteria can be used.

The procedure for using the staff hypsometer (Figure 1.7) is:

1. Tilt the tip towards your eye and straighten your arm. Grasp the staff loosely in your hand.
2. Allow the staff to swing to a vertical position, keeping your arm straight. You should now be holding the staff as far from its tip as your hand is from your eye.
3. Position yourself at a distance from the tree where it is just framed between the tip of the staff and your hand.
4. Measure or pace the slope distance to the tree. The horizontal distance you are away from the tree should be approximately equal to the height of the tree.

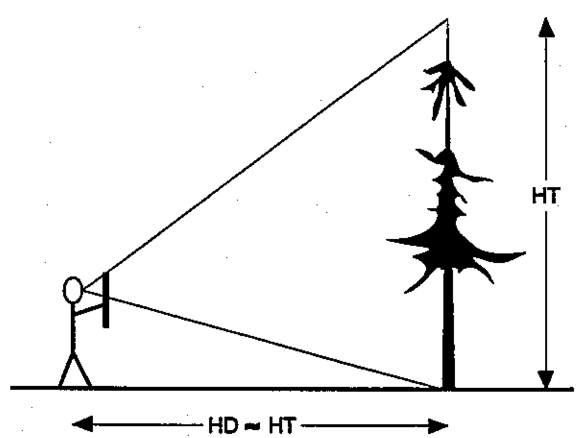


Figure 1.7 The principle behind the staff hypsometer.

The **Merritt hypsometer** is a special case of the staff hypsometer. It also consists of a straight pole or board; however, the Merritt hypsometer is calibrated so that the height of a tree can be read directly from the pole when you are a specific distance from the tree and holding the pole at a fixed distance from your eye. To use a Merritt hypsometer, you would stand an appropriate horizontal distance from the tree holding the pole vertically in front of you at the appropriate distance for the calibrations (e.g., your arm length, if that is how you calibrated it). You then would adjust your line of sight so that the zero point on the hypsometer coincides with the base of the tree. If you shift your sight to the top of the tree, the height of the tree can be read off the hypsometer at the point where your line of sight to the top of the tree intersects the pole. Like the staff hypsometer, the Merritt hypsometer provides only rough measurements of height.

Commercial varieties of the Merritt hypsometer are often sold on the back of Biltmore sticks. Like a Biltmore stick, it is easy to build and calibrate your own Merritt hypsometer. The formula for calibration is:

$$D = \frac{E \times \mathrm{HT}}{\mathrm{HD}}$$

where D is the distance on the hypsometer betyween the zero point and the mark for a particular tree height (HT); E is the distance the hypsometer is held from your eye (length of arm); HD is the horizontal distance you should stand away from the tree.

the trigonometric approach

The **trigonometric approach** is used most commonly. It involves measuring angles above and below the horizontal plane to the top and the bottom of the tree. The Suunto clinometer is frequently used to measure these angles. If you know the horizontal distance from the position where you make the angle measurements to the tree, you can calculate the height of the tree using trigonometric theory. Like the angle of slopes, these angles can be measured in either percentages or degrees. However, unlike correcting for slope, the sign of the angles is important. Any angle below the horizontal plane is considered negative; any angle above the horizontal plane is considered positive.

A good rule of thumb for selecting a spot from which to measure the angles to the top and bottom of the tree is to be about as far away from the tree as the tree is tall. If you are too much closer than this, the angle divisions on the scale of the measuring device you are using get quite close together, increasing the possibility of error. (As a general rule, angle measurements should be kept less than 100%.) If you are too much farther away, the visibility of the tree may be impaired. If the ground is sloping, measuring the angles from the uphill side allows you to use a shorter horizontal distance while still keeping the angles below 100%.

If HT is the height of the tree, H_1 is the height of the tree above the horizontal plane, and H_2 is the height of the tree below the horizontal plane, the formula for calculating tree height with the angles measured as percentages can be developed as follows. (Refer to Figure 1.8.)

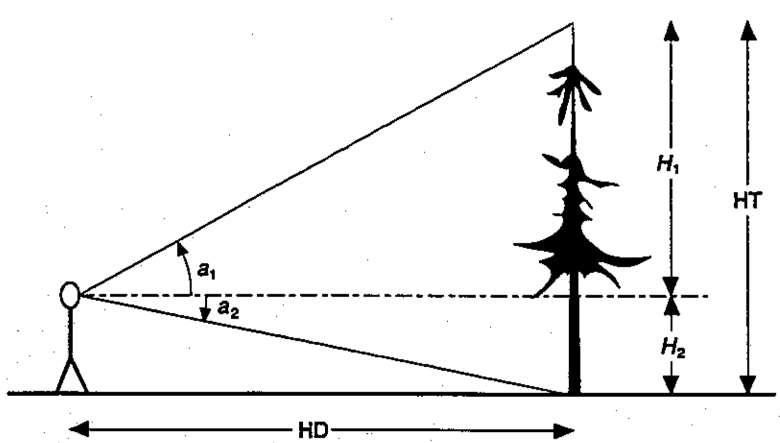


FIGURE 1.8 Diagram illustrating the trigonometric approach.

$$\begin{aligned}
\text{HT} &= H_1 + H_2 \\
H_1 &= \text{HD} \times a_1 \div 100 \\
H_2 &= -\text{HD} \times a_2 \div 100 \text{ (Recall that } a_2 \text{ is negative in Figure 1.3.)} \\
\text{HT} &= (\text{HD} \times a_1 \div 100) - (\text{HD} \times a_2 \div 100) \\
&= \text{HD} \div 100 \times (a_1 - a_2)
\end{aligned}$$

As an example, we will calculate the height of a tree that is 22 m (HD) away from where the angles were measured. The angle to the top of the tree (a_1) is 75%. The angle to the bottom of the tree (a_2) is –25%. The height of the tree is:

$$\begin{aligned}
\text{HT} &= 22 \div 100 \times (75 - (-25)) \\
&= 22 \div 100 \times (100) \\
&= 22 \text{ m}
\end{aligned}$$

We will use the same symbols to derive the formula for height when the angles are measured in degrees.

$$\begin{aligned}
\text{HT} &= H_1 + H_2 \\
H_1 &= \text{HD} \times \tan(a_1) \\
H_2 &= -\text{HD} \times \tan(a_2)
\end{aligned}$$

[Again, recall that a_2 is a negative angle in Figure 1.8.]

$$\begin{aligned}
\text{HT} &= \text{HD} \times \tan(a_1) - \text{HD} \times \tan(a_2) \\
&= \text{HD} \times [\tan(a_1) - \tan(a_2)]
\end{aligned}$$

To illustrate the use of this formula, we will use the same tree as in the previous example. Recall that HD was 22 m. The angle measurement to the top is 36.8°. The angle measurement to the bottom is –14°. The height of the tree is:

$$\begin{aligned}
\text{HT} &= 22 \times [\tan(36.8°) - \tan(-14°)] \\
&= 22 \times [0.748 - (-0.249]) \\
&= 22 \times (0.997) \\
&= 21.9 \text{ m}
\end{aligned}$$

As one last example, we will complicate things by introducing a slope and a slope distance. Assume the slope distance is 25 m and the slope angle (θ) is –10°. The angle to the tip of the tree (a_1) is 21.5° and the angle to the base of the tree (a_2) is –20.4°. Figure 1.9 illustrates this example. [It is often beneficial

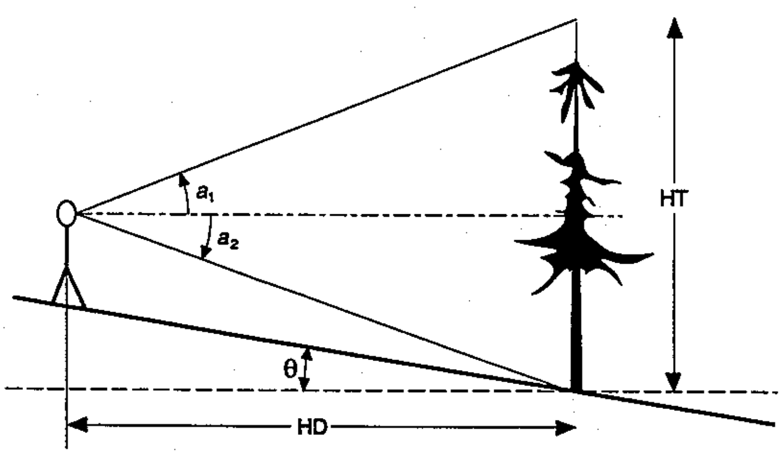


FIGURE 1.9 Drawing of the situation which occurs when you are measuring tree height on a down slope.

for you to draw a diagram when faced with a more complicated height calculation.] The first step is to calculate the horizontal distance:

$$\begin{aligned} HD &= SD \times \cos(\theta) = 25 \times \cos(10°) \\ &\text{[Note that the sign of the angle is not important.]} \\ &= 25 \times 0.985 \\ &= 24.62 \text{ m} \end{aligned}$$

Now we calculate the tree height as:

$$\begin{aligned} HT &= HD \times [\tan(a_1) - \tan(a_2)] \\ &= 24.62 \times [\tan(21.5°) - \tan(-20.4°)] \\ &\text{[The signs of the angles are important here.]} \\ &= 24.62 \times [0.394 - (-0.372)] \\ &= 18.86 \text{ m} \end{aligned}$$

Other instruments that can be used for measuring angles include the Haiga hypsometer and the Abney hand level. You will have a chance to work with these instruments during the on-campus laboratory session.

measuring the height of leaning trees

If at all possible, trees that are leaning should not be measured for height. If it is not possible to avoid measuring the height of these trees, the height above ground (HT') should be obtained from a direction perpendicular to the lean of the tree. One way to find the true height of the tree (HT) is by measuring the angle of the lean away from the perpendicular (θ) and applying trigonometry to find HT (Figure 1.10).

It follows from trigonometry that:

$$\sin(90° - \theta) = HT' \div HT$$

$$HT = HT' \div \sin(90 - \theta)$$

As an example, consider a tree with a measured height above ground (HT') of 30 m. (This can be calculated using the formulae that you learned in the previous section.) The angle of lean is 10°. The height of this tree is:

$$\begin{aligned} HT &= 30 \div \sin(90° - 10°) \\ &= 30 \div 0.985 \\ &= 30.46 \text{ m.} \end{aligned}$$

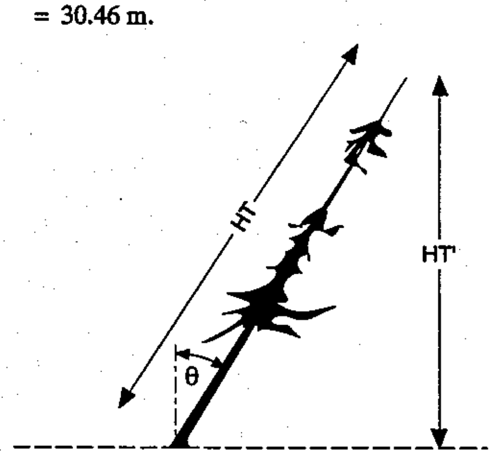


FIGURE 1.10 Variables of interest on a leaning tree.

MEASUREMENT OF UPPER STEM DIAMETERS

Upper stem diameters are defined as diameters which occur far enough up the stem of a tree as to be out of reach of an individual on the ground. Measurements of upper stem diameters are required occasionally for accurately measuring volumes for use in constructing volume or taper equations, for determining the merchantable point on a stem, or for determining form quotients. We will discuss these uses in Lesson 3, but the measurement procedure and examples of some of the instruments involved will be covered here.

Unless you choose to climb the tree or use a ladder, upper stem diameters must be obtained indirectly (i.e., at a distance). The process of measuring upper stem diameters indirectly can be logically broken into two parts: (1) determining the height of the point at which you are measuring; and (2) obtaining the measurement. We will describe each of these aspects separately.

Determining the Height of the Diameter Measurement

Determining the height above ground for any portion of the tree is not difficult using an instrument that measures angles, such as the Suunto clinometer. If you know the horizontal distance from the tree, then all you need to do is to determine the appropriate reading on the clinometer to correspond to the desired height. This is most easily done using the percentage scale. The procedure is:

1. Measure the horizontal distance you are away from the tree. If you are going to do the calculations in your head it is easiest if you choose an easy distance with which to work (e.g., 20 m as opposed to 19 m).
2. Obtain the percentage reading to the base of the tree.
3. Express the desired height as a percentage of the horizontal distance.
4. Add the percentages obtained in steps 2 and 3.

Say that you want to obtain an upper stem diameter at a point 15 m above the ground. You are 20 m (horizontal distance) from the tree. You take an angle measurement to the base of the tree and obtain a reading of –5%. The angle reading you should have at a height of 15 m would be:

$$\begin{aligned}\text{\% reading at 15 m} &= (15 \div 20) \times 100 + (-5)\\ &= 75 + (-5)\\ &= 70\end{aligned}$$

Obtaining an Indirect Measurement of Upper Stem Diameter

There are a number of instruments available that can be used for making upper stem diameter measurements. Four of the most common are: the Wheeler pentaprism dendrometer; the Barr and Stroud Optical Dendrometer; the Spiegel relascope; and the telerelascope. We will not describe the first two instruments. References for additional information are given in Avery and Burkhart. We will describe the relascope in some detail since it is a very versatile instrument and its use is relatively common. The telerelascope operates in much the same manner as the relascope, only it has telescopic sights. You will have an opportunity to practice using a relascope and a telerelascope during your on-campus lab session.

relascope

The **relascope** was invented by Walter Bitterlich in Austria around 1960. Bitterlich is also famous as being the main instigator behind the development of a technique called 'point sampling' (also known as 'prism cruising', 'angle count sampling', and so on). This technique will be described in considerable detail in Forestry 238. It is likely that the idea for the relascope evolved from his work with point sampling since some of the underlying principles are the same. The relascope can be used:

1. to measure tree heights indirectly as you would with a Suunto clinometer;
2. to measure tree heights indirectly with no calculations if you are an appropriate distance from the tree for one of the several height scales;
3. to fix angles for point sampling;
4. to find out how far you are away from an object (i.e., as a range finder);
5. to measure upper stem diameters.

After we describe the instrument, we will cover all the uses on the above list with the exception of item 3. This item is covered in the appropriate section of Forestry 238.

The relascope is approximately 12 cm high, 4 cm wide, and 6 cm deep. It operates somewhat like a Suunto clinometer in that there is a weighted wheel inside that rotates as the instrument is moved from the horizontal. However, there are many more scales on this wheel than on a clinometer. The wheel is housed inside a metal casing (Figure 1.11). There are three ground glass windows to provide light to the interior of the instrument. On one end there is an eyepiece for viewing the interior of the instrument; on the other there is a lens with a visor to adjust the light conditions. On the end with the lens, there is a brake button for locking and unlocking the weighted wheel. The wheel is locked when the button is in its normal position and unlocked when the button is pushed and held.

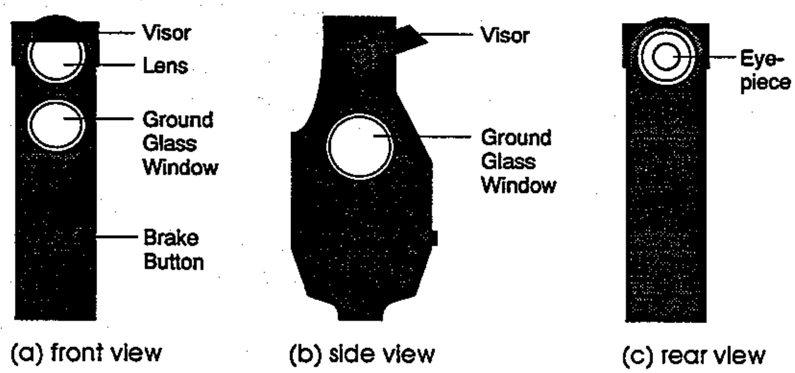


FIGURE 1.11 Drawing of a Spiegal relascope.

When you look through the eyepiece of the relascope you see a circular view (Figure 1.12). The circle is split in half horizontally. The top half is completely transparent and you are able to see the object that you wish to view (i.e., the bole

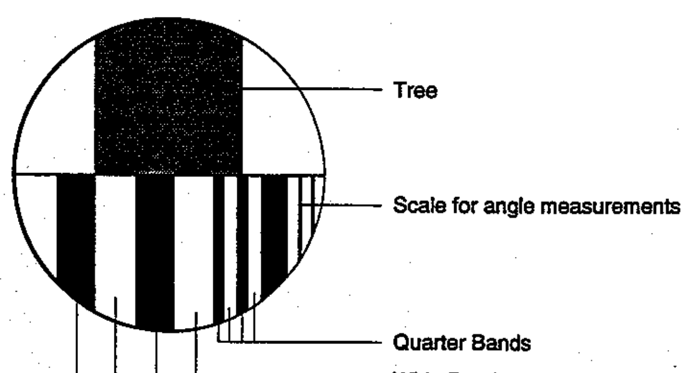


FIGURE 1.12 Simplified view through the eyepiece of a relascope.

of the tree). The bottom half is opaque and contains a number of alternating black and white contiguous bands. The majority of these bands are the same width at any given level for the instrument. These bands are widest when the relascope is pointed horizontally and taper uniformly as the relascope is moved up or down. At the right of the equal width bands are four narrower bands. Each of these bands equals one quarter of the other bands and are known as quarter bands. At the far right is a scale for measuring vertical angles in percentages or degrees.

Readings are made at the point where the opaque section meets the transparent section. In order to use the relascope, you sight at the portion of the tree on which you wish to make a reading while holding in the brake button. As soon as the scales stop moving, release the button and make a reading. The way you make a reading depends upon the kind of measurement you are making. We describe some of the measurements below.

measuring tree height

As indicated earlier, the relascope can be used to determine tree height in the same manner as you would with a Suunto clinometer. Also, you can determine height with fewer calculations if you are an appropriate distance from the tree. In order to determine height as you would with a clinometer, simply use the percentage or degree scale and follow the procedures we described for the clinometer. If you wish to determine height with fewer calculations, measure a horizontal distance from the tree equal to 4, 6, 8, ..., 20 m. The distance you select should be approximately equal to a visual estimate of the height. Once you are standing a known distance from the tree, choose the appropriate scale. (You can see which scale corresponds to your distance by looking at the scales while the relascope is held horizontally.) Take a reading at the top and bottom of the tree. The numbers you see will be the height above or below the horizontal in metres. When you subtract the bottom reading from the top reading, you will get the height of the tree. If you need to be at a greater distance from the tree than the largest scale, read from a scale corresponding to some fraction of the distance (e.g., $^1/2$) and multiply your readings by the inverse of that fraction (e.g., 2). For example, if you want to use a base distance of 32 m, you could read from the 16 m scale and multiply your readings by two. You could also choose to read from the 8 m scale and multiply your readings by four.

measuring diameter

Since the relascope measures the diameter indirectly (i.e., at a distance from the tree), you can use it to measure the diameter at any point along the bole. However, you should use a diameter tape or calipers when you can because these instruments are much more precise. The procedure for using the relascope to measure diameter is:

1. Measure the horizontal distance you are away from the tree.
2. Determine the height of the point on the tree at which you wish to make your measurement. (We described this procedure earlier.)
3. Determine the number of bands covering the diameter at that point. The easiest way to do this is to line up the left side of the tree with the beginning of a band in such a way that the right side falls into the region occupied by the quarter bands.
4. Convert the diameter reading in bands to a diameter in centimetres.:

$$\begin{aligned}\text{Diameter (cm)} &= \text{\# of bands} \times 0.02 \times \text{HD} \times 100\\ &= \text{\# of bands} \times 2 \times \text{HD}\end{aligned}$$

This procedure works because each full band has a width that exactly covers a 1 m width at a distance of 50 m. The tapering of the bands is such that it exactly matches the increase in sighting distance as the relascope sights above or below the horizontal. In other words, the relascope corrects for slope distance if the scale is allowed to rotate freely. One band will exactly cover a 1 m width at a horizontal distance of 50 m, no matter what the slope distance is to the point of the reading. This relationship is reflected in the 0.02 factor in the equation above. This factor is called the **relascope constant.** When you multiply the relascope constant times the horizontal distance in metres, you get the width of one band in metres. When you multiply this by 100, you get the band width in centimetres.

Consider a tree 20 m away (horizontal distance) from where you are standing. One relascope band would represent 20 x 0.02 x 100 which equals 40 cm. If you measure the diameter of the tree at some point as 1.25 bands using the relascope, the diameter at that point in centimetres would be 1.25 x 40 which equals 50 cm.

finding distance

The relascope can be used to determine the horizontal distance you are away from a target of known width. An instrument which performs this kind of measurement is known as a **range finder.** Although the relascope works fine as a range finder in theory, in practice imprecise width measurements cause estimated distances to be imprecise as well.

In order to use the relascope as a range finder, you need a target. The target is usually a 1 m stick or bar which is calibrated into 0.1 m bands of alternating colours (usually black and white). There is a fastener for attaching the target to a tree. To calculate the distance you are away from the target, you simply need to determine the width of target one or more relascope bands cover. If you allow the bands to move freely when sighting at the target, you will obtain a horizontal distance. If you lock the relascope bands at horizontal before sighting, you will determine a slope distance. Distance (D) may be calculated as:

$$\begin{aligned} D &= \text{band width} \div (0.02 \times 100) \\ &= \text{band width} \div 2 \end{aligned}$$

D will be in metres using this formula if band width is measured in centimetres.

Let's look at a few examples of how this formula can be used. Say you want to be 25 m away from the tree. What width of the target scale should one band cover? To answer this question, you simply need to substitute 25 for D and solve for band width.

$$\begin{aligned} D &= \text{band width} \div 2 \\ 25 &= \text{band width} \div 2 \end{aligned}$$

$$\text{band width} = 25 \times 2 = 50 \text{ cm}$$

Say you are an unknown distance from the target. You sight through the relascope allowing the bands to rotate. You note that two bands cover 70 cm on the target. What is the distance? The first step is to determine the width of one band. In this case it is 70 ÷ 2 which equals 35 cm. Then it is simply a matter of solving for D.

$$\begin{aligned} D &= \text{band width} \div 2 \\ &= 35 \div 2 \\ &= 17.5 \text{ m} \end{aligned}$$

Try this problem to see if you understand the process: You are standing at an unknown distance from a tree. A 1 m target is attached to the tree at eye level. When you sight at the target using a relascope with the scales locked at the horizontal position, you note that one band covers 45 cm. When you unlock the wheel, you note that one band covers 40 cm. What is the slope distance to the target? What is the horizontal distance? What is the percentage slope of the ground?

Answers: Slope Distance = 22.5 m
Horizontal Distance = 20.0 m
Percentage Slope = 51.5 %

MEASUREMENT OF OTHER TREE ATTRIBUTES

Measurements of other tree attributes are sometimes required. We will not cover many in this section, but we will briefly describe a few. You will have an opportunity to practice these measurements during the weekend laboratory session. We will also demonstrate the proper use of the instruments at that time.

BARK THICKNESS

Bark thickness measurements are used to convert diameters measured outside of the bark to diameters inside of the bark. When diameters are measured at heights easily accessible from the ground, bark thickness can be measured directly. Bark thickness can be quite variable around the circumference of a tree and usually needs to be measured a few times at different points and then averaged. The most frequently used instruments for directly measuring bark thickness are **bark gauges** and **increment borers.**

Bark gauges come in many forms. Generally, the gauge consists of a hollow metal bit that is driven through the bark of the tree, a scale indicating how far the bit has penetrated, and a handle for driving in the bit. The wood below the bark stops or severely retards the progress of the bit into the tree because it is considerably harder than the bark. This indicates to the user that the bark has been penetrated. Bark thickness is then read off the scale.

We discuss the use of an increment borer in the following section on "Age." An increment borer is generally used for measuring bark thickness if the bark is too thick for whatever bark gauge you have on hand or if measurements are also to be made on the wood portion of the core (e.g., ring width for determining diameter growth rates, age of the tree).

When bark thickness is required at points along the stem of a tree that are not easily accessible from the ground, it is usually approximated from the bark thickness at a more easily accessible height (e.g., breast height). A common assumption used in this approximation is that bark thickness represents a constant percentage of outside bark diameter at any point along the stem. Thus, the bark thickness at height h along the stem can be approximated as:

$$\mathrm{BT}_h = \frac{\mathrm{BT}}{\mathrm{dbh}} \times D_h$$

where BT is bark thickness at breast height and D_h is the outside bark diameter at the height at which bark thickness is required.

For converting outside bark diameters to inside bark diameters, bark thickness measurements need to be doubled. (Bark is found on both sides of the tree.) This quantity is called **double bark thickness** (DBT). Inside bark diameter at a height of h from the ground (DIB_h) is calculated from the outside bark diameter (D_h) and double bark thickness (DBT_h) at that height as:

$$\mathrm{DIB}_h = D_h - \mathrm{DBT}_h$$

Frequently, the two previous formulas are combined as:

$$\mathrm{DIB}_h = D_h - 2 \times \frac{\mathrm{BT}}{\mathrm{dbh}} \times D_h$$

AGE

Age at some height along the stem of trees growing in climates with one growing and one non-growing season within a year may be determined by counting the rings formed in the wood between the pith and the bark. Normally ring counts provide good indicators of age, but sometimes rings may be missing or extra rings may be present due to some form of stress (e.g., insect attack, disease, frost, drought) the tree may have suffered in the past. Also, rings on some species are difficult to see without a hand lens or microscope because of little colour differentiation between early and late wood or slow diameter growth rates.

Counting rings at any point along the stem can be done easily on felled and bucked trees (destructive sampling). If the trees are not to be sampled destructively, measurements are usually restricted to heights that can be easily reached from the ground. In this situation, an increment borer is used to extract a small round core (approximately 5 mm in diameter) of wood from the tree at some point on the tree's circumference. The core must include both the pith and the bark if it is to provide an accurate count of the tree's age.

Increment borers consist of a hollow tempered steel drill bit, a handle, and an extractor. They are available in different sizes. The bit will need to be at least as large as half of the outside bark diameter in order for the pith and bark to be included in the same core. The hollow bit is drilled into the tree at a direction thought by the user to intersect the pith. When the bit has penetrated to a sufficient distance to intersect the pith, the extractor is inserted into the bit, and the bit is rotated one complete revolution out of the tree to break off the core. The extractor is then removed from the bit along with the core. Finally, the bit is removed from the tree. If the core does not intersect the pith, another core will need to be taken to determine an accurate age.

Increment cores are frequently taken at breast height. If age is counted at that point, it is referred to as **breast height age.** If the **total age** of the tree is required, it can be measured at ground level or approximated by adding a correction factor to breast height age. This correction factor reflects the average number of years for a tree of that species to reach breast height on a specific site.

REVIEW/SELF-STUDY QUESTIONS

These questions should be answered before you go on to the graded assignment. *Do not submit answers to the tutor.* These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination.

1. What does mensuration entail?
2. Differentiate among precision, bias, and accuracy.
3. Briefly describe the five major sources of error.
4. Why can you not measure a 'tree'?
5. What is a variable?
6. What were the reasons for choosing breast height as a standard point for measuring diameters?
7. What is tree basal area?
8. How does a diameter tape work?
9. What is the most common error when using a diameter tape?
10. What are the relative advantages of calipers and diameter tapes?
11. Differentiate among total tree height, height to the living crown, and merchantable height.
12. How can height be directly measured? Why are these techniques not used all the time?
13. What is the relationship between horizontal distance and slope distance?
14. How is a staff hypsometer used? What is the underlying principle upon which the procedure is based?
15. What is the relationship among tree height, horizontal distance and percentage angles measured to the top and bottom of a tree?
16. What is the relationship among tree height, horizontal distance and angles measured in degrees to the top and bottom of a tree?
17. How can the height of leaning trees be determined?
18. What are upper stem diameters?
19. How is the height of an upper stem diameter measurement determined?
20. What are the different measurements that can be made with a Spiegel relascope?
21. How are diameters and distances measured using a relascope?

LESSON 2 MEASUREMENT OF TREE VOLUME

INTRODUCTION

LESSON OVERVIEW

The purpose of this lesson is to provide detail on principles relating to the measurement of tree volume. Common methods of estimating tree volume are covered in the next lesson. In this lesson, total stem volume, merchantable volume, and net volume are discussed.

LESSON OBJECTIVES

After studying this lesson and completing the self-study assignment, you should be able:

1. to define the notion of volume and to relate the volume of certain sections of the tree to the volume of particular geometric shapes;
2. to identify the strengths and limitations of Smalian's, Huber's and Newton's volume formulae;
3. to differentiate among total, merchantable, and net volume;
4. to state which measurements need to be made to calculate total, merchantable, and net volume quantities.

LESSON READINGS

The material in this lesson is not well covered in Avery and Burkhart, but is essential for obtaining a firm understanding of Lesson 3. The book by Husch, Miller and Beers (available through Extension Library loan) cover some of this material on pages 90-108.

LESSON ASSIGNMENT

There is no graded assignment for this lesson, but the material covered in this lesson will be required in order to complete the graded assignment for Lesson 3. Complete the self-study questions on page 33 at the end of this lesson.

WHAT IS VOLUME?

In order to manage the timber resource, foresters need to know the **volume** of the trees. The measurement of tree volume is therefore important. The volume of the tree can be restricted to the volume of the main stem or it may include the volume of the branches, roots and leaves. For this lesson, we will begin with a general discussion about volume, and then we will discuss how to measure the volume of the main stem and portions of the main stem.

Length, height, and width of an object are one-dimensional measurements; area of an object is a two-dimensional measurement; and volume is a three-dimensional measurement. Volume is therefore expressed in cubic units such as cubic metres (m^3) or cubic centimetres (cm^3).

Volume can be measured on an object by placing the object into a full container of water, collecting the water which is displaced by the object, and measuring the volume of this displaced water. Alternatively, for standard shapes, the volume of the object can be calculated using equations already derived. Volume of a cube or rectangular solid can be calculated by multiplying the length times the height times the width. The **volume of a cylinder** is found by multiplying the area at the base of the cylinder by the height of the cylinder ($A_b \times h = V$). The base of a cylinder is a circle; therefore the **area at the base** is calculated as:

$$A_b = \pi \times \frac{d^2}{4} = \pi r^2$$

where π is approximately 3.14159; d is the diameter of the circle and r is the radius of the circle (one-half of the diameter).

If diameter and height are measured in metres, then volume will be in cubic metres. For example, for a diameter of 0.20 metres and a height of 20 metres, the volume would be:

$$\text{volume} = 3.14159 \times \frac{0.20^2}{4} \times 20 = 0.6283 \text{ m}^3$$

EQUATIONS FOR VOLUME

The equations for volume of other shapes are summarized below.

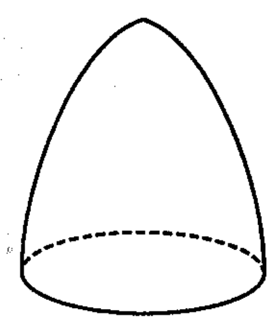


FIGURE 2.1 The paraboloid shape.

$$V = A_b \times \frac{h}{2}$$

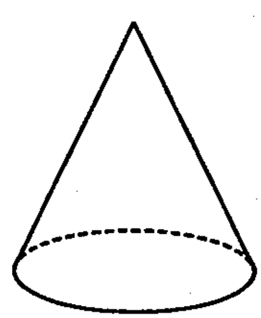


FIGURE 2.2 The cone shape.

$$V = A_b \times \frac{h}{3}$$

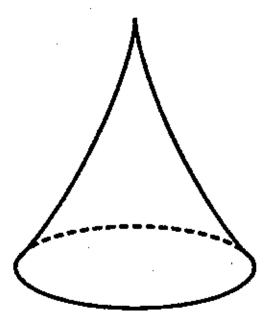


FIGURE 2.3 The neiloid shape.

$$V = A_b \times \frac{h}{4}$$

Shapes that have the same base area and height can be ranked from the least volume to the most volume, as neiloid, cone, paraboloid, and cylinder. By examining the equations for volume, we can get a feeling of what equations for other shapes might be. For instance, if the shape of the object is between a cylinder and a paraboloid, the equation for volume would still be area at the base times height, but the number that we divide by would be somewhere between 1 (for cylinder) and 2 (for the paraboloid).

The equations for calculating volume of these standard shapes were derived using calculus to find the **volume of a solid of revolution.** First, let's look at the cone shape. If we plot the radius of the cone-shaped object (y-axis) against the height of the object (x-axis), we obtain a graph that appears as in Figure 2.4.

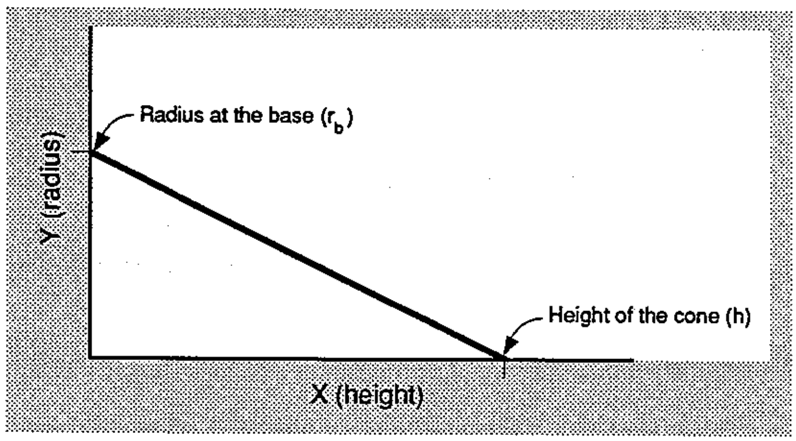


FIGURE 2.4 Radius versus height of a cone-shaped object.

We can also show this graph as the mirror image, following the cone from the top to the base as we go along the x-axis (Figure 2.5).

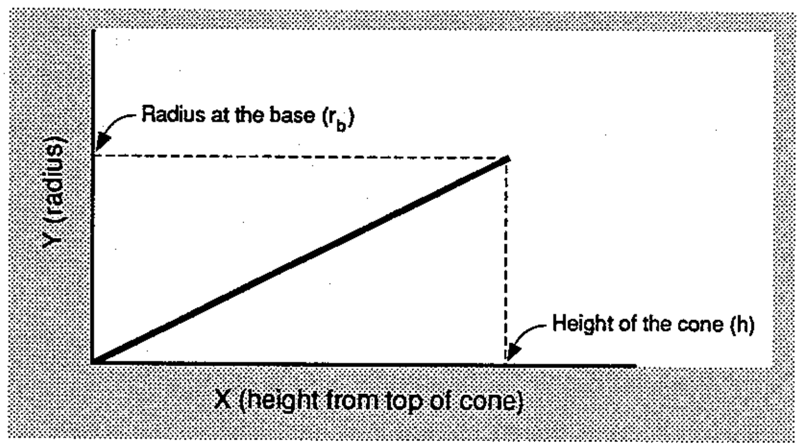


FIGURE 2.5 Radius versus height from the top of a cone-shaped object.

The relationship between the y-axis and the x-axis for any of the shapes that we have described (cylinder, paraboloid, cone, and neiloid) can be shown in equation form as follows:

$$y = c \times \sqrt{x^p} = \frac{r_b}{\sqrt{h^p}} \times \sqrt{x^p}$$

where x is height down the cone; y is radius at that height; c is a constant; p is a power; r_b is the radius at the base; and h is the height of the cone.

The power p varies depending on which shape we are interested in. For a cone, $p = 2$, therefore:

$$y = \frac{r_b}{\sqrt{h^2}} \times \sqrt{x^2} = \frac{r_b}{h} \times x$$

The square root signs disappear and we obtain an equation for the straight line shown in Figure 2.5, with a slope of r_b/h and an intercept of zero.

If we then imagine that the line that we have drawn on the graph is rotated in a circle around the x-axis (to obtain the third dimension needed for volume), we will see the cone shape emerging as in Figure 2.6.

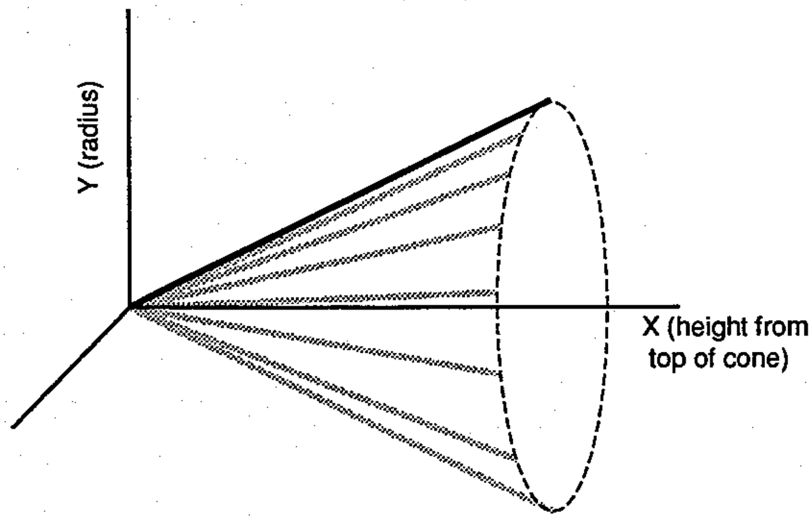


FIGURE 2.6 Rotation of radius versus height line

The term **volume of a solid of revolution** refers to the fact that a line is rotated around the x-axis (or curve if the shape is not a cone) and we are assuming that the shape that is formed by the revolution has volume (i.e., it is a solid).

If we cut a disk from the cone shape, we see that the disk looks much like a cylinder (Figure 2.7).

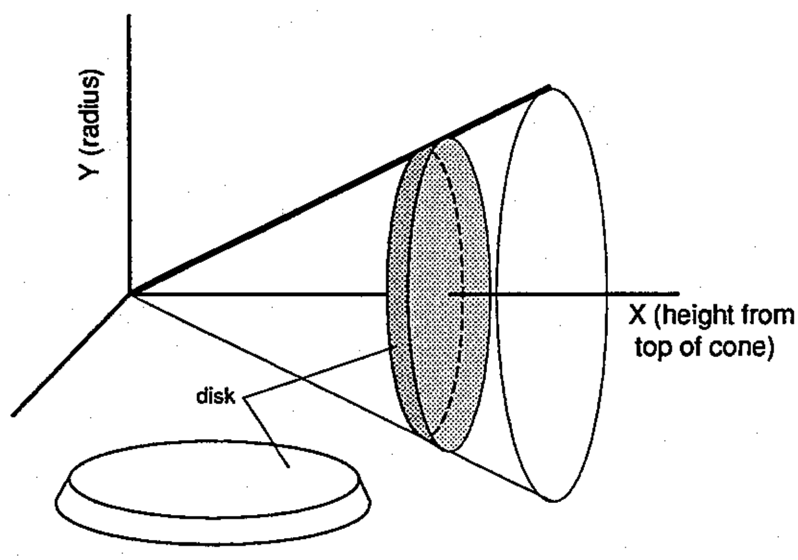


FIGURE 2.7 A disk from the cone appears much like a cylinder shape.

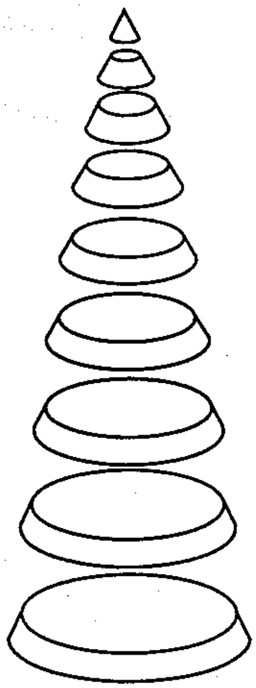


FIGURE 2.8 Volume of the cone as the sum of disks.

The volume of the disk is, approximately, that of a cylinder with a radius of y (defined in the equation above for the height down the cone from the top) and a disk height equal to a very small value, which we will call the dx. If we repeat this process for the entire cone shape, then the volume for each disk would be summed together to obtain the volume of the cone as shown in Figure 2.8. The volume of the cone is then the sum of the volumes for each cylinder with a height of dx and a radius equal to the y value at the small end of each disk.

Because each disk is not quite a cylinder in shape, we obtain a better calculation of volume if we let dx become a small value. The smaller the dx is, the closer the cylinder shape matches the shape of the disk taken from the cone. This is the process of **integration** used in calculus. When we use integration, we allow the height of each disk (the dx) to approach a very small value, close to zero. To calculate volume then, instead of dividing a standard shape (in this case the cone) into small disks, we can use calculus. The integration to obtain volume of the cone-shaped object is as follows. First:

$$\text{Area} = \pi r^2 = \pi y^2 = \pi \times \left(\frac{r_b}{h} \times x\right)^2 = \pi \times \left(\frac{r_b^2}{h^2}\right) \times x^2$$

Then:

$$\text{Volume} = \int_0^h (\text{area})\, dx = \int_0^h \left(\pi \times \frac{r_b^2}{h^2} \times x^2\right) dx$$

$$= \pi \times \frac{r_b^2}{h^2} \int_0^h x^2\, dx = \pi \times \frac{r_b^2}{h^2} \left. \frac{x^3}{3} \right|_0^h$$

$$= \pi \times \frac{r_b^2}{h^2} \times \left(\frac{h^3}{3} - \frac{0^3}{3}\right) = \pi \times r_b^2 \times \frac{h}{3}$$

$$= A_b \times \frac{h}{3}$$

Using integration, we have obtained the same equation for the cone given earlier in this lesson.

For the other standard shapes which we described, the power (p) changes, but the remainder of the equation to describe the radius (y) at different points down the shape (x), is unchanged, as shown in Table 2.1.

TABLE 2.1 Power (p) Values for Different Standard Shapes.

Shape	p	Equation
Cylinder	0	$y = \frac{r_b}{\sqrt{h^0}} \times \sqrt{x^0} = r_b$
Paraboloid	1	$y = \frac{r_b}{\sqrt{h^1}} \times \sqrt{x^1} = \frac{r_b}{\sqrt{h}} \times \sqrt{x}$
Cone	2	$y = \frac{r_b}{\sqrt{h^2}} \times \sqrt{x^2} = \frac{r_b}{h} \times x$
Neiloid	3	$y = \frac{r_b}{\sqrt{h^3}} \times \sqrt{x^3}$

The power (p) increases as the volume (for a given area at the base and height) decreases.

For the paraboloid shape, the graph of the radius from the top of the shape to the base of the shape appears as in Figure 2.9.

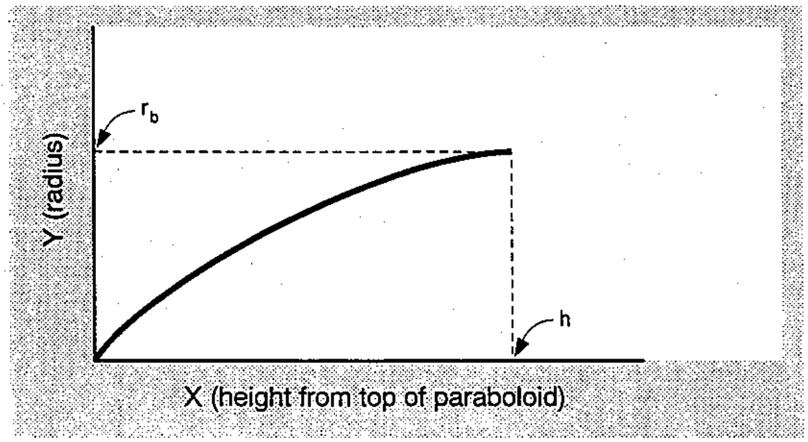


FIGURE 2.9 Radius versus height from the top for a paraboloid-shaped object.

Using integration to find the volume of a solid of revolution, the equation for a paraboloid shape is found by:

$$\text{Volume} = \int_0^h (\text{area})\,dx = \int_0^h \pi \times \left(\frac{r_b}{\sqrt{h}} \times \sqrt{x}\right)^2 dx$$

$$= \pi \times \frac{r_b^2}{h}\int_0^h x\,dx = \pi \times \frac{r_b^2}{h}\left|_0^h \frac{x^2}{2}\right.$$

$$= \pi \times \frac{r_b^2}{h^2} \times \left(\frac{h^2}{2} - \frac{0^2}{2}\right) = \pi \times r_b^2 \times \frac{h}{2}$$

$$= A_b \times \frac{h}{2}$$

This, again, is the familiar equation given earlier in this lesson. You may wish to try using integration to verify the equation for volume assuming the neiloid shape.

If the shape of the object is not one of the standard shapes (e.g., a cone) we could still find the volume of the object if we knew the equation which describes the relationship between the radius (y) and the height down from the top of the object (x). Once this equation is known, integration can be used to calculate volume.

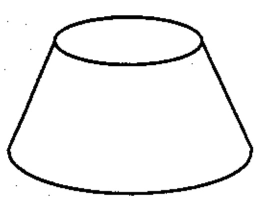


FIGURE 2.10 The cone frustum shape.

Integration can also be used to find the volume for a part of the object, if we know the equation for radius (y) with height (x). For instance, if we remove the top of any of the standard shapes, we are left with the base of the shape. This is called the **frustrum**. Notice that a frustum of a cylinder will also be a cylinder. A **cone frustum** is shown in Figure 2.10.

In this case, we would follow the same procedure as for the entire shape, but we replace the *0* as the lower limit of the integration with some other limit, and we would replace the *h* of the upper limit with some other limit. For example, if the object were a cone 15 metres in height, we could find the volume from 2 metres down from the top to 15 metres (i.e., remove the top of the cone), as follows:

$$\text{Volume} = \int_2^{15} (\text{area})\, dx = \int_2^{15} \pi \times \left(\frac{r_b}{h} \times x\right)^2 dx$$

$$= \pi \times \frac{r_b^2}{h^2} \int_2^{15} x^2\, dx = \pi \times \frac{r_b^2}{h^2} \Big|_2^{15} \frac{x^3}{3}$$

$$= \pi \times \frac{r_b^2}{h^2} \times \left(\frac{15^3}{3} - \frac{2^3}{3}\right)$$

Since $h = 15$, we simplify this to:

$$\text{Volume} = \frac{A_b}{15^2} \times \left(\frac{15^3}{3} - \frac{2^3}{3}\right)$$

If the diameter of the base is 18.0 cm, the area at the base is 0.0254 m^2, and the volume for this cone frustum would be:

$$\frac{0.0254}{15^2} \times \left(\frac{15^3}{3} - \frac{2^3}{3}\right) = 0.1269\ \text{m}^3$$

MEASUREMENT OF TOTAL TREE VOLUME

We have discussed in general the idea of what volume is and how volume can be calculated for standard shapes. However, we are most interested in tree volume.

The volume of the main stem of a tree (**total tree volume**) can be expressed as volume **inside bark (i.b.)** or less commonly, as volume **outside bark (o.b.)**. If the volume i.b. is needed, then all measurements of the diameter or radius of the tree stem must also be i.b., or must be corrected from o.b. measurements to i.b. by using a measure of bark thickness (discussed in Lesson 1).

There are several ways of measuring total tree volume. Each of these methods has inherent advantages and disadvantages.

Measuring the Volume of Water Displaced

The volume of a tree could be measured by cutting the tree down, placing it into a large container, and measuring the volume of the water displaced. The instrument which is used to measure tree volume in this way is called a **xylometer.** The disadvantages to using this instrument to measure volume are: the measurement is time consuming; the instrument (and the tree) is bulky; and the tree must be cut down (and cut into logs if the tree is large). Also, if volume i.b. is required, the tree bark must be removed before the tree is immersed, or an allowance for the volume of the bark must be made. The advantages to using this instrument are: an accurate measurement of volume is obtained; and the volume of parts of the tree outside the main stem can be measured.

Assuming a Standard Shape for the Tree

An alternative to using a xylometer is to assume that the tree shape is one of the standard shapes. We can then do some simple measurements and calculate tree volume. For instance, we could assume that the tree shape is a cone shape. To calculate volume, we must have a measurement of the diameter of the tree at the base to calculate the area at the base, as well as a measurement of the tree height. The volume is found by entering the area at the base and the height of the tree into the equation to calculate the volume of a cone.

For example, for a tree of 14.35 cm diameter i.b. at the base (0.1435 m) and a total height of 16.8 m, the volume, assuming a cone shape would be:

$$\frac{\left(\pi \times \frac{(0.1435)^2}{4} \times 16.8\right)}{3} = 0.0906\ \mathrm{m}^3$$

Note that volume i.b. is calculated if the diameter at the base is i.b., and volume o.b. is calculated if the diameter at the base is o.b. The diameter o.b. can be measured using one of the instruments described in Lesson 1, such as a diameter tape or calipers; if volume i.b. is needed, the diameter i.b. can be found by reducing the diameter o.b. by two times the thickness of the bark (**double bark thickness**). Another way of obtaining diameter i.b. is to cut down the tree and measure the diameter i.b. on the cut base using a ruler or measuring tape.

The advantages of using this technique to obtain tree volume are: only a few simple tree measurements are needed; the tree can remain standing; and the required calculations are simple to perform. However, this method does not usually result in an accurate measurement of total tree volume.

Assuming Standard Shapes for Different Parts of the Tree

Instead of assuming one standard shape for the whole stem, different shapes may be used for different parts of the tree. If we draw a diagram of the main stem of a typical tree, we can get an idea of which shapes to use for which part of the tree stem (Figure 2.11).

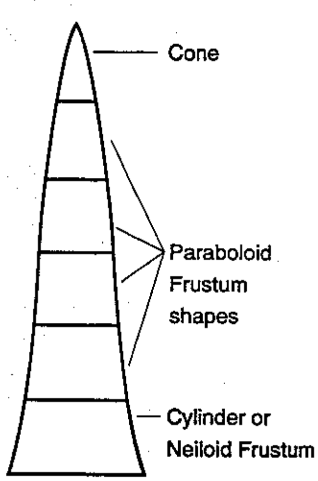


Figure 2.11 Standard shapes assumed for portions of the tree bole.

The part of the main stem nearest the ground looks like either a neiloid frustum or a cylinder. If the main stem of the tree is divided into sections, these sections generally have a paraboloid frustum shape. The tree top appears to be a cone shape.

In order to calculate total tree volume, we will need measurements of the diameter at the top and base of each section of the tree and the length of each section. We can obtain these measurements using a relascope, as described in Lesson 1 of this course. (Note that this will give us diameter o.b., which must be corrected to diameter i.b. to calculate volume i.b.) Alternatively, we can cut the tree down, cut the tree stem into sections, and measure the diameter at the top of each section (i.b. or o.b., depending on which volume is wanted) using a ruler, and measure the length of each section.

Once we have the tree measurements, we could calculate the volume of each section using integration, as discussed in the previous section. However, instead of using integration, we will use equations already derived for the standard shapes and frustums of the standard shapes.

Husch and others list several different equations for calculating volume of frustums of the standard shapes (page 101). For this course, we will cover only three of these equations.

1. **Smalian's equation** for a paraboloid frustum

$$V = (A_b + A_u) \times \frac{h}{2}$$

2. **Huber's equation** for a paraboloid frustum

$$V = A_m \times h$$

3. **Newton's equation** for a neiloid, cone or paraboloid frustum

$$V = (A_b + 4A_m + A_u) \times \frac{h}{6}$$

where A_b is the area at the base of the tree section;
A_m is the area at the middle of the tree section;
A_u is the area at the top of the tree section;
h is the height (or length) of the section of the tree.

Note that if the shape is a cylinder, all of these equations simplify to the equation given previously for calculating volume of a cylinder.

Newton's formula is used to calculate the volume of the neiloid, cone, or paraboloid frustum shapes exactly. However, a measurement of the diameter at the midpoint of the section is needed, as well as the diameter measurements at the top and base of the tree section. Not only is an additional diameter measurement needed, but you also must be able to locate the midpoint of the section in order to measure this diameter. This is particularly difficult to do if the tree sections are stacked logs in which only the base and top of the log are easy to see and measure.

Smalian's and Huber's formulas are exact only if the tree section follows the paraboloid frustum shape. If the true shape of the section is closer to a cone or neiloid frustum shape, then Smalian's formula will overestimate the volume of the section and Huber's formula will underestimate the volume. Overestimation or underestimation is minimized if the lengths of the sections (or logs) are smaller than 2.5 metres. Because Smalian's formula requires only a measurement of the diameters at the base and top of the tree section (or log), as well as the height of the section, this formula is most often used. Huber's equation requires only one diameter measurement at the middle of the section or log, but as already mentioned for Newton's formula, this diameter measurement is often difficult to obtain.

Smalian's formula for a paraboloid frustum can be thought of as the average area of the section, times the height of the section (average area = $(A_b + A_u)/2$). This is the same as calculating the volume of a cylinder with the area equal to the average area. Another way of interpreting Smalian's equation is to rewrite the equation as follows:

$$V = \left(A_b \times \frac{h}{2}\right) + \left(A_u \times \frac{h}{2}\right)$$

The volume of the paraboloid is then the sum of two volumes. First, the volume of a cylinder with the area equal to A_b and the height equal to $h/2$ is calculated.

Next, the volume of a cylinder with the area equal to A_u and the height equal to $h/2$ is calculated. The volume from these two cylinders is then added to obtain the volume of the paraboloid frustum.

An example calculation of the total volume of a tree is shown in Table 2.2. In this example, the equation for a cone is used to calculate volume for the top section, the equation for a cylinder is used for the bottom section, and Smalian's equation is used for the intermediate sections.

TABLE 2.2 Calculation of Tree Volume.

Section No.	d.i.b. at top	Length	Area	Volume
1 (Stump)	14.35	0.30	0.0162	0.0049
2	14.00	1.50	0.0154	0.0237
3	12.40	1.50	0.0121	0.0206
4	11.45	1.50	0.0103	0.0168
5	11.35	1.50	0.0101	0.0153
6	10.35	1.50	0.0084	0.0139
7	8.35	1.50	0.0055	0.0104
8	7.55	1.50	0.0045	0.0075
9	6.20	1.50	0.0030	0.0056
10	4.80	1.50	0.0018	0.0036
11 (Top)	0.00	3.00	0.0000	0.0018
Totals		**16.80**		**0.1241**

Notice that the volume i.b. using this method is 0.1241 m^3, whereas using a cone shape for the entire tree stem results in a volume i.b. of 0.0906 m^3.

The advantages of using this method as a measurement of tree volume are: as with the second method, measurements can be taken on the standing tree if desired; the measurements are relatively simple; and the method results in a more accurate measurement than the second method shown, although more diameter and height measurements are required.

MEASUREMENT OF MERCHANTABLE TREE VOLUME

Merchantable tree volume is defined as volume of the merchantable part of the tree stem. Generally, the **merchantable limits** (or merchantable standards) of the tree are defined as a **lower height limit** (stump height), referring to the point where the tree is cut above ground when harvested, and an **upper diameter limit**, usually diameter inside bark, referring to the smallest usable diameter. Limits commonly used in B.C. are a lower limit of 0.30 or 0.45 metres stump height, and an upper limit of 10.0, 15.0, or 20.0 cm top diameter i.b.

Another criterion of merchantability which may be added is a minimum length required between the stump height and the minimum top diameter i.b. (**merchantable length**). For instance, we may decide that a tree with less than 2.5 metres between the lower and upper merchantability limits is not usable for the product of interest. In this case, if we encounter a tree which has less than 2.5 metres, the merchantable volume is considered zero. Other criteria of merchantability exist in Canada, and vary between provinces. For this reason, we will concentrate on the measurement of merchantable volume defined by a lower and an upper limit only. As with the measurement of total tree volume, there are several methods which we can use to measure merchantable tree volume.

MEASURING THE VOLUME OF WATER DISPLACED

To measure the merchantable tree volume, only the merchantable part of the tree stem is placed in the xylometer (see page 27). The volume of the water displaced is then a measure of merchantable volume. This is the most accurate measurement of merchantable tree volume, but it has all the disadvantages noted for the measurement of total tree volume using the xylometer.

ASSUMING A STANDARD SHAPE FOR THE MERCHANTABLE PART OF THE TREE

The merchantable part of the tree can be assumed to be a frustum of a standard shape. If we have measurements of the diameter for the tree stem at the lower limit, a measurement of the diameter at the upper limit, and a measurement of the merchantable length, we can calculate volume for the merchantable part of the tree stem using Smalian's equation. For example, for a diameter i.b. at a stump height of 0.30 metres of 40.0 cm (0.40 m), an upper limit defined as 20 cm diameter i.b., and a merchantable length of 10 m, the volume can be calculated as follows:

$$A_b = \left(\frac{\pi \times 0.40^2}{4}\right) = 0.1257\ \text{m}^2$$

$$A_u = \left(\frac{\pi \times 0.20^2}{4}\right) = 0.0314\ \text{m}^2$$

$$\text{Volume} = \frac{(0.1257 + 0.0314)}{2} \times 10.0 = 0.786\ \text{m}^3$$

The advantage of this method is that few tree measurements are required. However, there is potential for inaccuracy. Because the equations given for frustums of shapes (e.g., Smalian's equation) result in biased measures of volume for long lengths (greater than 2.5 metres), the merchantable volume calculation is inaccurate for trees that are not the shape assumed for the equation.

ASSUMING STANDARD SHAPES FOR DIFFERENT PARTS OF THE TREE

To increase the accuracy in measuring merchantable volume, measurements of diameter and length can be taken for each section along the main stem. As for total tree volume measurements of a standing tree, the measurements can be taken using a relascope, resulting in diameter o.b. measures. Alternatively, the tree can be felled and cut into sections. In each case, the volume for each section can be calculated using an equation for a frustum shape. Smalian's equation is most commonly used.

The disadvantages of this method of measurement are: it is not as accurate as using a xylometer; and many tree measurements must be taken. Also, accurate i.b. measurements of diameter can be obtained only by felling the tree and the tree must be cut into short lengths of 2.5 metres or less.

MEASUREMENT OF NET MERCHANTABLE TREE VOLUME

The merchantable tree volume, defined in the previous section, can also be called **gross merchantable tree volume**. Conversely, **net merchantable tree volume** is gross merchantable tree volume less the amount of wood which is not usable for the product of interest. Wood that is unusable includes decayed wood which is structurally damaged and the remainder of the wood in a decayed log.

Definitions of unusable wood vary depending on the product of interest, and between provinces in Canada, even for the same product. In B.C., three deductions are made to gross merchantable volume to obtain net merchantable volume. These are **decay, waste,** and **breakage.** ***Decay*** is defined, generally, as wood that can be pierced by a dull object such as your fingernail or a pencil. ***Waste*** is defined as the remaining volume if the log is more than 50% decayed. Also, if the merchantable part of the tree has more than 50% decay volume, the remaining volume in the tree is waste. ***Breakage*** is the volume lost because the tree breaks when it impacts the ground upon felling.

The measurement of net merchantable tree volume can be done by:
- processing the tree into the product and measuring the resulting volume.
- using scaling techniques.

The first technique is used in assessing the return in processing and will not be further discussed in this course. The scaling technique, commonly used to obtain net merchantable volume for stacked logs, is discussed in Lesson 4.

REVIEW/SELF-STUDY QUESTIONS

These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. In general, how is the volume of any object accurately measured?
2. What would be the volume of a neiloid-shaped object with a radius of 12.0 cm and a height of 4.5 m? (Answer: 0.0509 m^3)
   If a paraboloid-shaped object had the same radius and height, would it have more or less volume?
3. Why is the use of calculus to find volume of a regularly shaped object called the 'volume of a solid of revolution'?
4. If the shape of an object appeared to be between a cone and a neiloid, what equation might you expect for volume?
5. What is a xylometer used for?
6. Name three techniques for measuring total tree volume and list the advantages and disadvantages of each.
7. If the shape of a section of the tree is a paraboloid frustum, which of the three equations, Newton's, Smalian's, or Huber's, can be used to calculate volume of the section exactly?
8. How does Smalian's equation for a paraboloid frustum relate to the equation for calculating the volume of a cylinder?
9. What is merchantable tree volume? merchantable limits?
10. What is the difference between gross merchantable tree volume and net merchantable tree volume?
11. What three deductions are made to gross merchantable tree volume in B.C.? Describe each of them.

LESSON 3 ESTIMATION OF TREE VOLUME

INTRODUCTION

LESSON OVERVIEW

The purpose of this lesson is to show how to estimate total, merchantable, and net merchantable tree volume from the measurements of other related tree attributes, using linear regression techniques. A review of simple linear regression is presented, as well as a description of the measures of tree form. Commonly used equations are presented. Taper functions are introduced.

Methods to measure tree volume were discussed in Lesson 2. Because the accurate measurement of tree volume is time consuming, and we often wish to know the volume of many trees, tree volume is commonly estimated from other tree attributes which are more easily measured.

For this lesson, we will begin with a discussion about regression, a technique used to fit equations, with an emphasis on simple linear regression. Next, because measures of form may be used in the estimation of tree volume, a section on these measurements is presented. The estimation of total tree volume is then covered, followed by the estimation of merchantable tree volume, and of net merchantable tree volume. Finally, a discussion of taper functions is presented.

LESSON OBJECTIVES

After studying this lesson and completing the assignment, you should be able:

1. to show how regression can be used to obtain estimates of tree attributes and interpret the results of regression;
2. to contrast different measures of tree form;
3. to differentiate between local, standard, and form class total tree volume functions;
4. to show an understanding of the ideas behind estimating merchantable tree volume from total tree volume by using merchantable ratio functions and by using reduction equations;
5. to describe the principles used in estimating decay reductions used in Canada and the U.S. and the decay, waste, and breakage functions used in B.C.;
6. to describe the principles behind taper functions.

LESSON READINGS

Relevant information on this topic can be found in Avery and Burkhart, pages 23–32, 111–114, and 120–139.

LESSON ASSIGNMENT

Answer the self-study questions at the end of this lesson before you complete Graded Assignment #2 in Appendix A which you submit to your tutor with a pink assignment sheet. You should send it by the date indicated on your course schedule.

REGRESSION TECHNIQUES FOR FITTING EQUATIONS

Linear regression is used to relate one dependent variable (y) to independent variables (x's). For multiple linear regression, more than one independent variable is used. This regression procedure will be described in the second mensuration course (Forestry 238). For simple linear regression, there is only one independent variable and the relationship between x and y is described as:

$$y_i = \beta_0 + \beta_1 \times x_i + \varepsilon_i$$

where β_0 is the y intercept;
β_1 is the slope, or change in y for the change in x;
ε_i is the difference between the observed y value, and the value given by the line;
β_0 and β_1 are called the coefficients of the regression line.

Simple linear regression is used to find estimates of the slope and intercept from sample data. Once these estimates are obtained, we can:
- determine how well the regression line fits the sample data (goodness-of-fit);
- calculate confidence intervals for the true slope and intercept (population);
- calculate confidence intervals for a mean predicted y value (y value on the regression line);
- test whether the regression is significant.

ASSUMPTIONS OF SIMPLE LINEAR REGRESSION

In order to estimate the slope and intercept, calculate confidence intervals for the coefficients and for the predicted line, and test for significance, some assumptions concerning the sample data must be met.

1. The relationship between x and y is linear. If this assumption is not met, then the regression line will not fit the data well. For example, if the plot of y on x shows a curvilinear trend as in Figure 3.1, then the fitted regression line will appear as shown in Figure 3.2.

For the smaller and larger x values, the y values will be overestimated. For the middle range of x, the y values will be underestimated. In this case, another equation is needed, such as using x^2 instead of x as the independent variable. Alternately, multiple linear regression may be used if both x and x^2 are included in the equation.

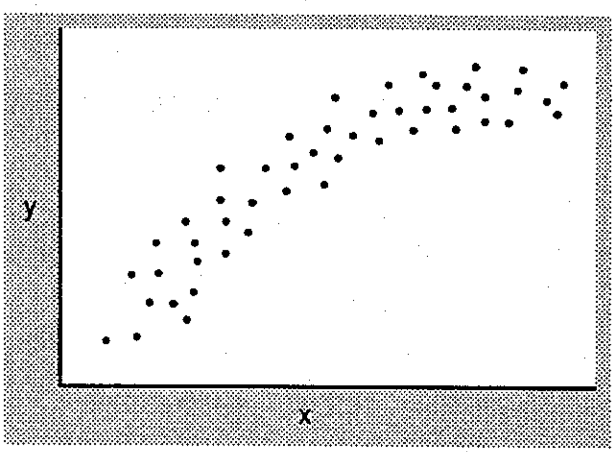


FIGURE 3.1 Curvilinear trend between x and y.

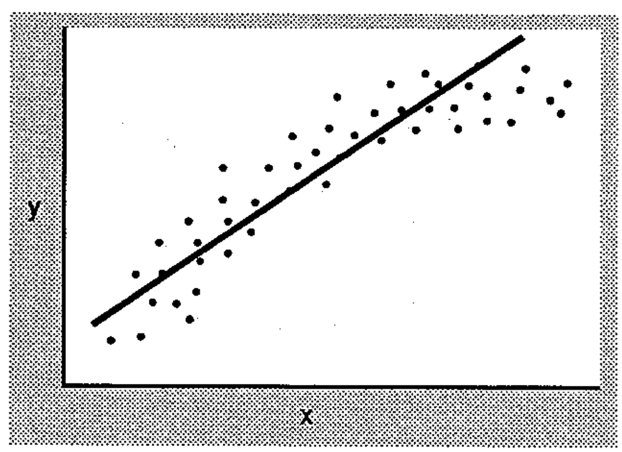


FIGURE 3.2 Linear regression for a curvilinear trend.

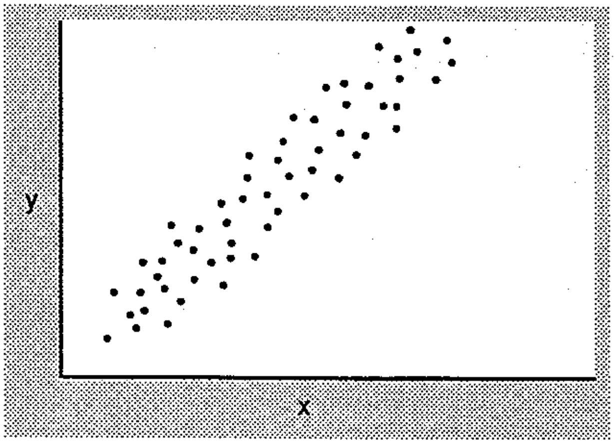


FIGURE 3.3 Equal variances of y values.

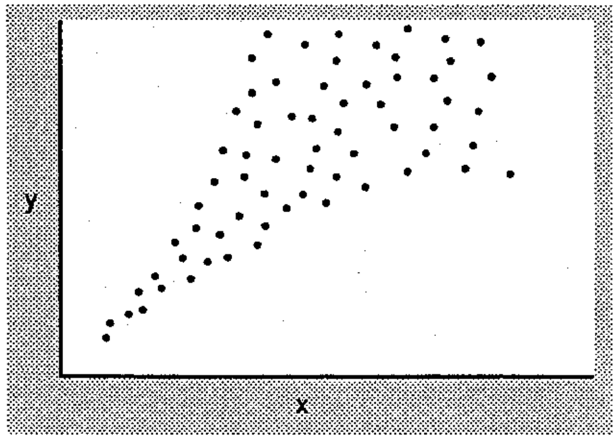


FIGURE 3.4 Unequal variances of y values.

2. The variance of the y values must be the same for every x value. If this assumption is met, then the graph of y versus x will appear as in Figure 3.3. If the variance of y differs across the range of the x, then the graph of y versus x will appear as in Figure 3.4.

The estimated coefficients (slope and intercept) will be unbiased, but the usual estimates of the variances of these coefficients will be biased. The result is that we cannot calculate confidence intervals nor can we test the significance of the regression line. We can, however, estimate the coefficients of the regression line and calculate the goodness-of-fit. In this case, a technique called "weighted least squares regression" should be used if confidence intervals are wanted.

3. Each observation (x,y) must be independent of all other observations. For example, if we take a measurement of dbh and height of a tree at age 30 years and again when the tree is 50 years, the two sample observations are related (i.e., they are not independent). In this case, if we were to use simple linear regression, the estimated coefficients would be unbiased (as with the unequal variances discussed above), and we could calculate how well the line fits the data (goodness-of-fit). However, we could not calculate confidence intervals nor test hypotheses. The confidence intervals would be narrower than expected. If confidence intervals were desired, the difference in the y variable between time periods could be used rather than y itself. Similarly, the difference in the x variable between time periods would be used as the independent variable.

4. The y values must be normally distributed for each x value. For a given range of the x value (e.g., dbh from 15 to 20 cm), the plot of the y values should appear as in Figure 3.5.

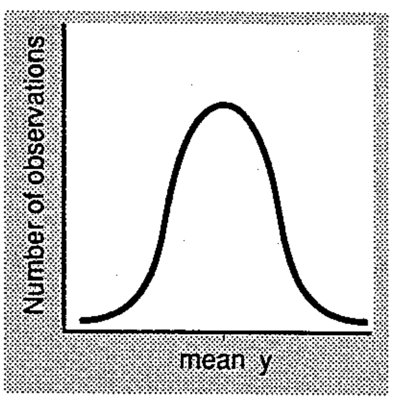


FIGURE 3.5 Normal distribution of y values for a given x value.

The pattern in Figure 3.5 is called the **normal distribution** because it occurs in many natural situations. The majority of the y values are at the centre (mean value) and a few are at the upper and at the lower ends. If the y values are not normally distributed, confidence intervals and significance tests will be inaccurate. For most forestry problems, the assumption of normal distribution is met. The exception is when the y values are recorded as percentages, such as percent of insect infestation. In this case, we can modify simple linear regression by transforming the y values (this transformation is called the arcsine transformation) and using simple linear regression for the transformed data.

5. The x values must be measured without error. The assumption in regression is that the x values are fixed. If this is not true, then the estimates of the coefficients and of their variances will be biased. The estimated regression line, confidence intervals, and significance tests will all be inaccurate. For most forestry applications, this assumption is met if measurements are taken carefully.

6. The y values are selected randomly for each x value. Ideally, the x values are fixed (Assumption 5) and for each fixed x value, a list of all possible y values is made. A random selection of a sample of the y values is then obtained. For most forestry applications, because this list is difficult to obtain, this assumption is usually not met and is commonly ignored.

Estimating the Slope and Intercept

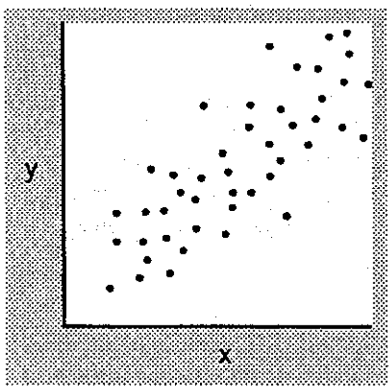


Figure 3.6 Plot of x versus y.

If we take a sample set of data and plot the y values versus the x values we obtain a graph like that shown in Figure 3.6. We would like to find the line which best describes the relationship between x and y. One way that we could do this is to guess where the line should be, and then add up the differences between the observed y value and the y value from the regression line over all sample points. If our guessed line is balanced, then the sum of the differences (called **error terms** or **residuals**) will add up to zero; the negative error terms will balance the positive error terms. In fact, any line which goes through the point described by $\bar{x}$ (the mean value for x) and $\bar{y}$ (the mean value for y), will have a sum of error terms equal to zero. The best line, then, cannot be based on just a balanced line.

The criterion used in linear least squares regression is that the sum of the squared error terms is minimized. This will also result in a balanced line which goes through the point defined by $\bar{x}$ and $\bar{y}$. We could try various lines until we find one which will give us this result. Alternatively, we can use calculus to find a quick solution.

We would like to find an estimate of the slope and of the y intercept, using the sample data, which results in a minimum for the sum of squared error terms. In calculus, if we have an equation, we can take the first derivative of the equation with respect to the unknown value, set the derivative to zero, and solve for the unknown value which will be either a minimum or a maximum point. However, in this case, we wish to solve for two unknown values. If we had two equations, we could solve for these two unknowns.

To obtain two equations, we first identify the equation for which we want to find the minimum:

$$S = \text{sum of squared error terms}$$
$$= \sum e_i^2 = \sum (y_i - \hat{y}_i)^2$$
$$= \sum \left(y_i - (b_0 + b_1 \times x_i)\right)^2$$

where y_i and x_i are the observed values for the sample;
$\hat{y}_i$ is the predicted value for the sample; $\hat{y}_i = b_0 + b_1 \times x_i$;
e_i is the difference between the observed value for y and the predicted value from the regression line (residual).

We then take partial derivatives. Since we have two unknown values, b_0 and b_1, we will have two partial derivatives as follows:

$$\frac{\delta S}{\delta b_0} = -2\sum(y_i - b_0 - b_1 \times x_i)$$

$$\frac{\delta S}{\delta b_1} = -2\sum x_i(y_i - b_0 - b_1 \times x_i)$$

If we set these equations to zero, solve for b_0 in the first equation, and for b_1 in the second equation, after some rearranging we obtain:

$$b_0 = \bar{y} - b_1 \times \bar{x}$$

$$b_1 = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{\text{SPXY}}{\text{SSX}}$$

where $\bar{y}$ is the mean of y for the sample;
$\bar{x}$ is the mean of x for the sample.

SPXY is the sum of product XY, and is calculated as:

$$\text{SPXY} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

SSX is the sum of squares X, and is calculated as:

$$\text{SSX} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

SPXY can be a negative or a positive value. SSX must be positive because the differences are squared. The second form of the equation for SPXY and for SSX is used more commonly than the first because the calculations are easier.

By inputting the x and y values from the sample data, we will find the values for b_0 and b_1 which will result in minimizing the sum of squared differences.

The following example shows the fit of an equation which is used to estimate height of a tree given the dbh of that tree. Ten trees were measured and used to find an estimate of the slope and intercept of this equation. These values are shown in Table 3.1.

To verify the assumptions of simple linear regression, a plot of the y values versus the predicted x values, as shown in Figure 3.7, is needed.

The relationship between dbh and height for this limited range of data appears to be linear (*Note:* If we had data for the lower dbh values, the relationship may appear to be not linear). Because we have only ten samples, it is difficult to see whether the variance of height is the same for every dbh. There does not seem to be any reason to disagree with this assumption for these ten points, however. The remaining three assumptions will also be considered to be met, because the measurement of dbh and height for one sample tree can be considered independent of the measurement of dbh and height on other sample trees, dbh can be

TABLE 3.1 Measurements of dbh and Height of Ten Trees

Tree No.	dbh (cm) (x_i)	Height (m) (y_i)	x_i^2	y_i^2	x_iy_i
1	23.7	16.20	561.7	262.44	383.94
2	24.7	16.45	610.1	270.60	406.32
3	17.9	14.90	320.4	222.01	266.71
4	20.5	14.43	420.2	208.22	295.82
5	16.5	13.93	272.2	194.04	229.84
6	15.8	13.90	249.6	193.21	219.62
7	12.8	13.10	163.8	171.61	167.68
8	13.7	13.60	187.7	184.96	186.32
9	12.5	11.82	156.2	139.71	147.75
10	11.4	9.55	130.0	91.20	108.87
Totals	**169.5**	**137.88**	**3071.9**	**1938.00**	**2412.87**

$$\bar{x} = \frac{169.5}{10} = 17.0; \quad \bar{y} = \frac{137.88}{10} = 13.79$$

$$\text{SPXY} = 2412.87 - \frac{169.5 \times 137.88}{10} = 75.80$$

$$\text{SSX} = 3071.9 - \frac{(169.5)^2}{10} = 198.88$$

$$b_1 = \frac{75.80}{198.88} = 0.3811$$

$$b_0 = 13.79 - (0.3811 \times 17.0) = 7.311$$

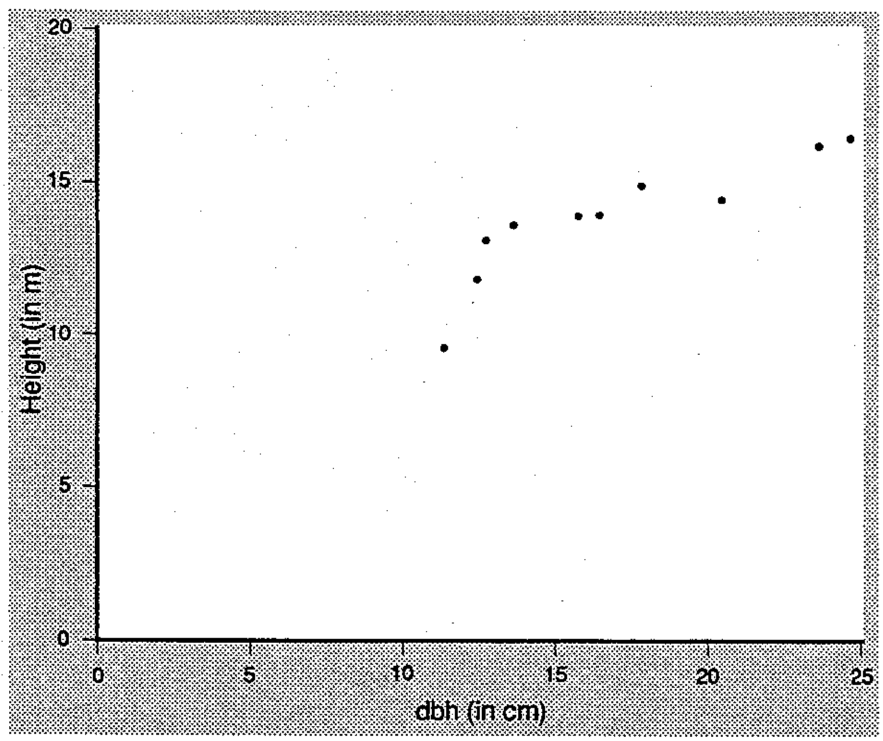


FIGURE 3.7 Plot of dbh (x) versus height (y).

considered to be measured without error (as long as the measurements were taken with care), and the distribution of height for any dbh can be considered normal. We can then say that the estimated regression line is unbiased, and we can calculate confidence intervals and test hypotheses for this regression line.

GOODNESS-OF-FIT

Once we have found the line which best describes the relationship between y and x, we can also find out how well the line fits the sample data (**goodness-of-fit**). There are two measures which are commonly used to describe the goodness-of-fit. These are the **coefficient of determination** and the **standard error of the estimate.**

The coefficient of determination (r^2 value) represents the amount of variation of the y observations accounted for by the regression. The calculation of r^2 is as follows:

$$r^2 = \frac{\text{SSR}}{\text{SSY}} = 1 - \frac{\text{SSE}}{\text{SSY}}$$

where SSY is the total sum of squares, or the sum of squares of y, calculated as:

$$\text{SSY} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$$

SSE is the sum of squares for the error. This is the term that we are minimizing. SSE is calculated as follows:

$$\text{SSE} = \sum (y_i - \hat{y}_i)^2 = 1 - \text{SSR}$$

SSR is calculated as follows:

$$\text{SSR} = \sum (\hat{y}_i - \bar{y})^2 = b_1 \times \text{SPXY}$$

Notice that SSY, SSE, and SSR must all be positive, because we are summing squared differences. As with SPXY and SSX, the second form of these equations is most commonly used because the calculations are simpler.

Once SSR and SSY are calculated, the r^2 value can be found. For the height-dbh equation given in the example, the r^2 value is:

$$\text{SSY} = 1938.0 - \frac{(137.88)^2}{10} = 36.91$$

$$\text{SSR} = 0.3811 \times 75.80 = 28.89$$

$$r^2 = \frac{28.89}{36.71} = 0.79$$

The total sum of squares (SSY) is a sum of the SSR and the SSE. As SSE decreases, SSR rises, and so the r^2 value rises. Since SSR can never be negative and can never be greater than the SSY value, the r^2 value must be between zero and one. A higher r^2 value indicates that a high proportion of the variation in y is accounted by the regression.

The standard error of the estimate gives us an indication of how far the observations are spread around the regression line. Approximately 68% of the sample observations will be within one standard error of the estimate on either side of the regression line. Approximately 95% of the sample observations will be within two standard errors from the regression line, as shown in Figure 3.8.

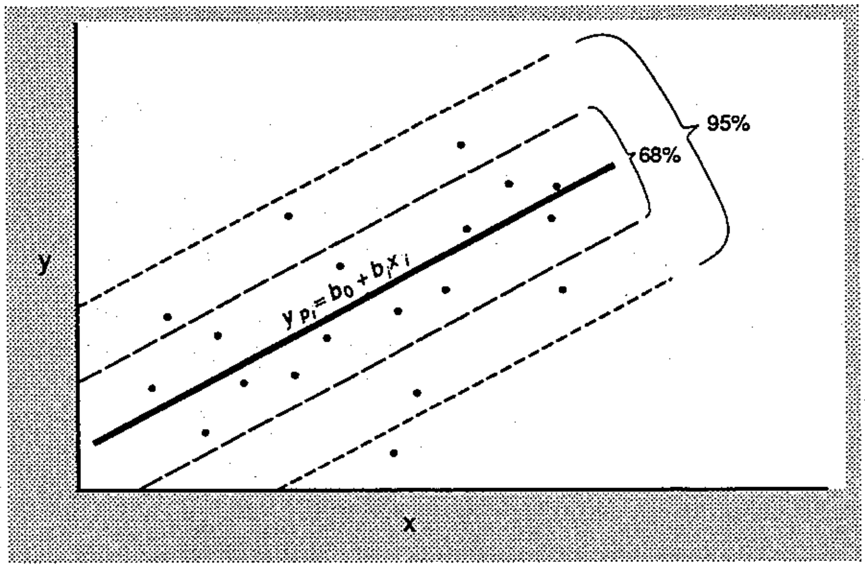


FIGURE 3.8 Standard error lines around the regression line.

The standard error of the estimate is calculated as follows:

$$SE_E = \sqrt{MSE} = \sqrt{\frac{SSE}{n-2}}$$

The term $n-2$ represents the **degrees of freedom.** Two degrees of freedom are lost from the n degrees of freedom from the sample points, because the intercept and the slope are fixed values for the fitted regression line.

For the height-dbh example, the standard error of the estimate is:

$$SSE = SSY - SSR$$
$$= 36.91 - 28.89 = 8.02$$

$$SE_E = \sqrt{\frac{8.02}{8}} = 1.00$$

Thus, approximately 68% of our sample observations are within 1.00 metres above or below the height-dbh regression line. Approximately 95% of our sample observations are within 2.00 metres above or below the regression line.

CONFIDENCE INTERVALS FOR THE COEFFICIENTS AND THE PREDICTED VALUES

The estimated slope and intercept were based on sample data. Because the sample set is only a subset of the **population** (all of the observations that we could measure), we would like to know how likely it is that if we took another sample set, we would obtain similar estimates of the slope and intercept. If we could repeat the sampling, taking n samples each time, for a large enough value of n (greater than 30 or so), we would find that the values that we obtained for the slope would have a frequency distribution as shown in Figure 3.9.

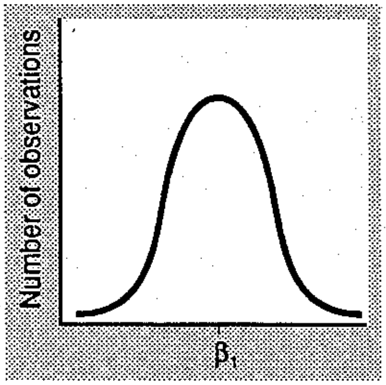


FIGURE 3.9 Distribution of estimated slope values calculated from samples of size *n*.

Most of the values would be similar to the true population parameter for the slope (if we could measure all samples and calculate the slope), and the mean of all of the slope values that we would calculate would be equal to the population value for the slope. We would see a similar pattern for the estimated intercept values.

Because of the tendency of the values that we might calculate for the intercept and for the slope to be normally distributed, we can calculate confidence intervals for the true population intercept and slope using the information from only one sample set. We generally use the t-distribution instead of the normal distribution, because this results in wider, therefore more conservative, confidence intervals. Also, the t-distribution is used when the standard deviations are not known. Since we do not know the true standard deviations of the slope and intercept estimates, these must be estimated from the sample data.

For the intercept value, the confidence interval is calculated as follows:

$$b_0 \pm t_{(n-2,1-\alpha/2)} s_{b_0}$$

which is simply the estimated intercept plus or minus the value in the t table (for n-2 degrees of freedom, and $1-\alpha/2$ probability, where α is the significance level) times the standard deviation of the estimated intercept. The standard deviation of the intercept (how much the estimates would be expected to vary for different sample sets) is calculated as follows:

$$s_{b_0} = \mathrm{SE_E}\sqrt{\left(\frac{1}{n}\right)\left(\frac{\sum x_i^2}{\mathrm{SSX}}\right)}$$

Using the height-dbh example, a 95% confidence interval for the intercept is:

$$s_{b_0} = 1.00\sqrt{\left(\frac{1}{10}\right)\times\left(\frac{3071.9}{198.88}\right)} = 1.24$$

$$b_0 = 7.311; \quad t_{8,0.975} = 2.306$$

$$7.311 \pm 2.306 \times 1.24 = (4.45, 10.17)$$

For the estimated slope coefficient, the confidence interval can be calculated as follows:

$$b_1 \pm t_{(n-2,1-\alpha/2)} s_{b_1}$$

where s_{b_1} is the standard deviation of the estimated slope coefficient and is calculated as:

$$s_{b_1} = \mathrm{SE_E}\sqrt{\frac{1}{\mathrm{SSX}}}$$

For the height-dbh equation, the 95% confidence interval for β_1 is:

$$s_{b_1} = 1.00\sqrt{\frac{1}{198.88}} = 0.071$$

$$b_1 = 0.3811; \quad t_{8,0.975} = 2.306$$

$$0.3811 \pm 2.306 \times 0.071 = (0.217, 0.545)$$

A confidence interval can also be calculated for the mean predicted value of y for a given value of x. As with the estimated slope and intercept values, the distribution of the predicted y values at any point on the x-axis follows a normal distribution. That is, if we were to repeat the sampling over and over again, the distribution of the predicted y values for any point on the x-axis would be a bell-shaped, normal distribution. We can therefore use the information from a single sample set to calculate a confidence interval for the true value of the population regression line (if we had measured all observations and fitted the regression line). The calculation for the mean predicted y value, given a particular x value is as follows:

$$\hat{y}|x_0 \pm t_{(n-2,1-\alpha/2)} s_{\hat{y}|x_0}$$

where $\hat{y}|x_0$ is the value of y given that x is fixed at the value x_0;

$$\hat{y}|x_0 = b_0 + b_1 \times x_0.$$

The standard deviation of the value of y given that x is fixed at the value x_0 is represented by $s_{\hat{y}|x_0}$. This is calculated as:

$$s_{\hat{y}|x_0} = \mathrm{SE_E}\sqrt{\left(\frac{1}{n}\right) + \frac{(x_0 - \bar{x})^2}{\mathrm{SSX}}}$$

If we calculate (1-α) % confidence intervals (e.g., if α is 0.05, then we calculate 95% confidence intervals), for the mean value of y all along the x-axis, the resulting graph of the confidence intervals would appear as in Figure 3.10.

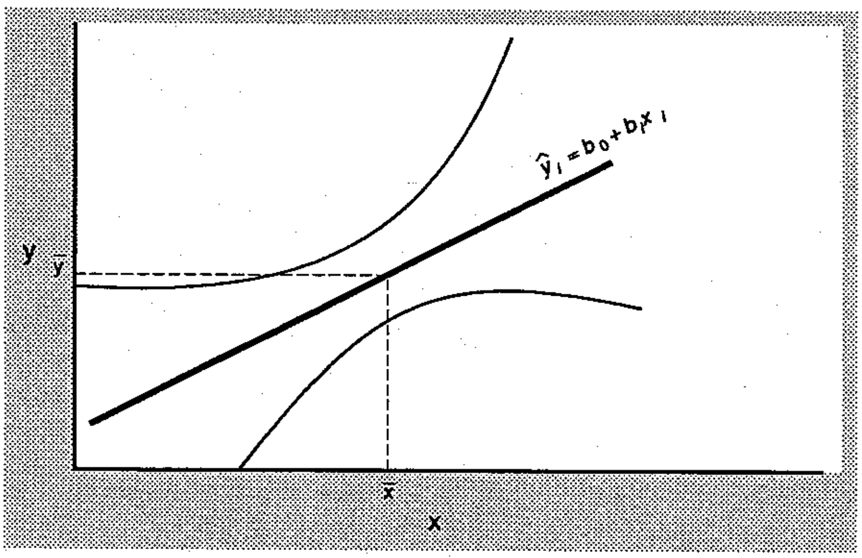


FIGURE 3.10 Confidence bands along the regression line.

This does not appear the same as Figure 3.8. Figure 3.8 gives only a range for the sample observations, whereas Figure 3.10 gives the confidence intervals for the true value of the regression line. Because the regression line must pass through the point defined by $\bar{y}$ and $\bar{x}$, the regression is more fixed at this point. There is still some variation in the level of the line, because we have the sample mean of y and of x, not the population mean values. However, as we go farther away from this central point on the regression line, the confidence intervals become wider. The regression line can vary not only in level but also in slope if we go away from the mean values (Figure 3.11).

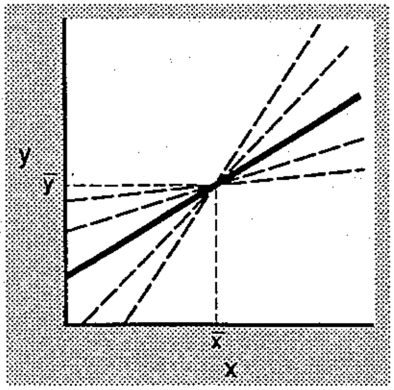


FIGURE 3.11 Variability of the estimated regression line.

For the height-dbh regression line, we can calculate a 95% confidence interval for the mean predicted y value as shown below for a dbh of 20.0 cm ($x_0 = 20.0$).

$$s_{\hat{y}|x_0} = 1.00\sqrt{\left(\frac{1}{10}\right) + \frac{(20.0 - 17.0)^2}{198.88}} = 0.3811$$

$$\hat{y}|x_0 \text{ (height at dbh} = 20) = 7.311 + 0.3811 \times 20.0 = 14.93$$

$$t_{8,0.975} = 2.306$$

$$14.93 \pm 2.306 \times 0.3811 = (14.05, 15.81)$$

The mean predicted height for a dbh of 20 cm will be between 14.05 to 15.81 metres for 95% of all possible samples of size n that we might take from the population.

SIGNIFICANCE OF THE REGRESSION

The **significance of the regression** is a term which means, "Is the relationship of x and y important?" Basically, if the true slope coefficient is zero ($\beta_1 = 0$), then the line showing the relationship between x and y is a horizontal line. This means that y does not change, no matter what the x value is, and the relationship is not significant. In other words, there is no statistical relationship between x and y.

To test whether or not a regression line is significant, we test whether or not the true slope coefficient is zero, as in the following hypothesis statement.

$$H_0\text{: } \beta_1 = 0$$
$$H_1\text{: } \beta_1 \neq 0$$

There are several ways in which we can test this hypothesis statement. First, we can calculate a confidence interval for β_1. If the confidence interval does not include zero, the null hypothesis is rejected, the true slope coefficient is likely not zero, and the regression is significant. For the height-dbh regression line, the confidence interval for β_1 was (0.217, 0.545). Since this does not include zero, the regression is significant.

Another test which we can perform is the ***F*-test**. This tests the same hypothesis again. The test statistic must first be calculated:

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{\left(\frac{\text{SSR}}{1}\right)}{\left(\frac{\text{SSE}}{(n-2)}\right)}$$

This is then compared to the critical value from an F-distribution table for one degree of freedom for the numerator (across the top of most tables), n–2 degrees of freedom for the denominator (down the side of most tables), and $(1-\alpha)$ probability, where α is the significance level. If the test statistic is larger than the tabular value, we reject the null hypothesis (H_0) and conclude that the regression is significant (true slope likely not zero). For the height-dbh regression line, the test statistic for the F-test is:

$$F = \frac{(28.89)}{\left(\frac{8.028}{8}\right)} = 28.79$$

Since the tabular value for the F distribution with 1 and 8 degrees of freedom and 95% probability ($F_{1,8,0.95}$) is 5.32, the test statistic is greater than critical value and the regression is significant. Notice that we obtained the same results by calculating the confidence interval for β_1.

Another way of testing significance is to calculate the **correlation coefficient** (r value). This is found by taking the square root of the r^2 value. The r value is then compared to critical values from an r-distribution table. If the r value is greater than the critical value, the regression is significant.

Finally, the significance of the regression can be found by using a ***t*-test**. The test statistic for the t-test is as follows:

$$t = \frac{(b_1 - c)}{s_{b_1}}$$

The hypothesized value, c, in this case is zero. The critical value is the value from the t-distribution for n-2 degrees of freedom and ($1-\alpha/2$) probability. If the t statistic calculated is greater than the critical value from the t table, or is less than the negative of the critical t value, the regression is significant.

Testing Hypothesis Statements Concerning the Regression Coefficient

The t-test described in the previous section may be used to test other hypothesis statements about the slope by putting in another value for c. The resulting test statistic would be again compared to the critical value from the t-distribution table for n–2 degrees of freedom and ($1-\alpha/2$) probability. The hypothesis would be rejected if the t statistic were greater than the critical t value.

Similarly the t-test could be used to test the intercept by using the following test statistic:

$$t = \frac{(b_0 - c)}{s_{b_0}}$$

This could be used to test if the intercept were zero, for example ($c = 0$). The t statistic would be compared to the critical t value (as defined above for testing the slope). If the test statistic is greater than the critical t value, or is less than the negative of the critical t value, the null hypothesis (H_0: $\beta_0 = c$) would be rejected.

Weighted Least Squares and Conditional Regression

If the assumption that the variance of the y values is constant for every x value is not met, we can use simple linear regression to obtain estimates of the slope and intercept. However, we cannot calculate confidence intervals for the true slope and intercept, nor can we test whether or not the regression is significant. We need an alternative technique to fit the regression line in this case. The alternative that we will discuss is called **weighted least squares.**

Weighted least squares is like simple linear regression, except that we transform the x and y values, and calculate coefficients and confidence intervals for the transformed data. For forestry applications, often the variance of the y values increases with the value of the x values, as shown in Figures 3.12a and 3.12b.

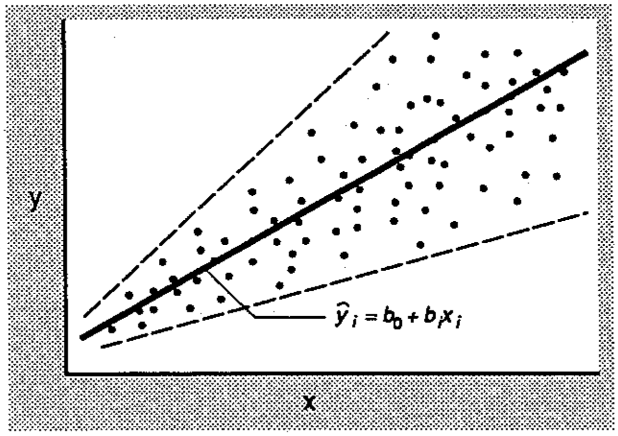


FIGURE 3.12(a) Variance of y proportional to x.

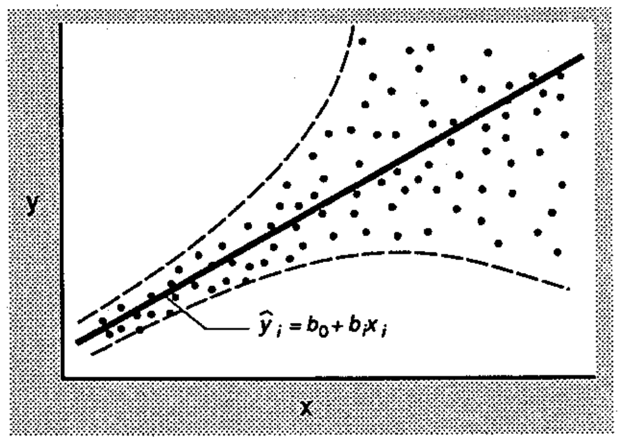


FIGURE 3.12(b) Variance of y proportional to x^2.

The weighted least squares procedure in this case, is to divide the x and y values by the square root of x if the variance looks like Figure 3.12a, and by x if the variance looks like Figure 3.12b. Once the x and y values are transformed, we proceed with the simple linear regression using the transformed data. An example of this technique will be shown in the "Estimating Total Tree Volume" section of this lesson.

With **conditioned regression**, we have some additional information about the slope or the intercept, and we would like to restrict the estimates of the slope or intercept based on this information. A common restriction is that the intercept should be zero. This means that when x is zero, y should also be zero. However, a conditioning of the intercept should be considered only if the confidence interval for the intercept from the unconditioned regression contains the fixed value.

The simple linear regression technique is modified slightly for conditioned regression through a y-intercept of zero.

$$b_1 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$b_0 = 0.0$$

$$r^2 \text{ (cond.)} = \frac{\text{SSR (cond.)}}{\text{SSY (cond.)}} = \frac{b_1 \times \sum x_i y_i}{\sum y_i^2}$$

$$\text{SSE (cond.)} = \text{SSY (cond.)} - \text{SSR (cond.)} = \sum y_i^2 - b_1 \times \sum x_i y_i$$

$$\text{SE}_\text{E} \text{ (cond.)} = \sqrt{\text{MSE (cond.)}} = \sqrt{\frac{\text{SSE (cond.)}}{(n-1)}}$$

Since we are assuming that the true intercept (β_0) is zero, we know what it is and therefore we do not calculate a confidence interval. For β_1, the true slope, the confidence interval is calculated as:

$$b_1 \pm t_{(n-1,1-\alpha/2)} s_{b_1}$$

where $s_{b_1} = \text{SE}_\text{E}\ (\text{cond.})\sqrt{\dfrac{1}{\sum x_i^2}}$

The degrees of freedom are *n–1* for the conditioned regression because only the slope is being estimated (degrees of freedom are *n–2* when the slope *and* intercept are estimated).

An an example for the height-dbh regression, we might expect that the *y* intercept would be 1.3 m since the height of the tree is 1.3 m when the dbh is zero. If we fix the intercept at 1.3 metres, we obtain:

$$\text{height}_i = 1.3 + \beta_1 \times \text{dbh}_i + \varepsilon_i$$

$$\left(\text{height}_i - 1.3\right) = \beta_1 \times \text{dbh}_i + \varepsilon_i$$

To obtain an estimate of the slope using the sample data, we let *y* equal the height minus 1.3, and we condition the regression so that the intercept for this reduced height is zero. The information for the conditioned regression is shown in Table 3.2.

TABLE 3.2 Measurements of dbh and height of ten trees, modified for conditioned regression.

Tree No.	dbh (cm) (x_i)	Height–1.3 m (y_i)	x_i^2	y_i^2	x_iy_i
1	23.7	14.90	561.7	220.01	353.13
2	24.7	15.15	610.1	229.52	374.20
3	17.9	13.60	320.4	184.96	243.44
4	20.5	13.13	420.2	172.40	269.16
5	16.5	12.63	272.2	159.52	208.40
6	15.8	12.60	249.6	158.76	199.08
7	12.8	11.80	163.8	139.24	151.04
8	13.7	12.30	187.7	151.29	168.51
9	12.5	10.52	156.2	110.67	131.50
10	11.4	8.25	130.0	68.06	94.05
Totals	**169.5**	**124.88**	**3071.9**	**1594.43**	**2192.51**

$$\bar{x} = \frac{169.5}{10} = 17.0; \quad \bar{y} = \frac{124.88}{10} 12.49$$

$$b_1 = \frac{2192.51}{3071.9} = 0.7137; \quad b_0 = 0.0$$

$$r^2\ (\text{cond.}) = \frac{0.7137 \times 2192.51}{1593.43} = \frac{1564.79}{1594.43} = 0.9814$$

$$\text{SE}_\text{E}\ (\text{cond.}) = \sqrt{\frac{1594.43 - 1564.79}{9}} = \sqrt{\frac{29.64}{9}} = 1.815$$

The fitted regression equation is as follows:

$$(\hat{height}_i - 1.3) = 0.0 + 0.7137 \times dbh_i$$

$$\hat{height}_i = 1.3 + 0.7137 \times dbh_i$$

The r^2 value for the conditioned regression ***cannot*** be compared to the r^2 value for the unconditioned regression. The SE_E value can be compared, and is higher for the conditioned regression, because we are not allowing the regression line to be as flexible. Also, the estimated slope is higher for the conditioned regression because we are forcing the regression line through 1.3 when the x value is zero. For the unconditioned regression, the confidence interval for the intercept was 3.76 to 10.86, values higher than 1.3. We have, therefore, "tipped" the regression line up by introducing a fixed intercept at a value smaller than the lower limit of the confidence interval, causing an increase in the estimated slope. We have used this height and dbh data to illustrate the procedures for conditioned regression. However, the conditioning of this regression of height versus dbh is ***not*** justified because the confidence interval for the intercept did not include 1.3.

MEASUREMENT OF TREE FORM

The height and dbh measurement of the tree define the tree size, whereas form measures provide more information about tree shape. Tree form varies by species, and also by age, dbh, height, and stand attributes such as density. The common form measurements can be divided into two types: form factors and form quotients.

FORM FACTORS

A form factor is a ratio of the volume of the tree in comparison to the volume of a standard shape. For instance, we may use the volume of a cylinder with the same area at the base and height as a reference point, as follows:

$$f = \frac{\text{volume of the tree}}{\text{volume of a cylinder}}$$

This particular form factor is called the **cylindrical form factor**. This measurement is useful as a form measurement, but it is difficult to obtain. We must have an accurate measurement of the tree's volume, either by using a xylometer or, as described in Lesson 2, by dividing the tree into sections and obtaining measurements of each section as described in Lesson 2. The second method for measurement described in Lesson 2 — assuming one standard shape for the entire tree stem — would not be accurate enough for a form factor.

FORM QUOTIENT

The use of a form quotient is perhaps a more practical measurement of form. Basically a form quotient is a ratio (or it may be expressed as a percent) of an upper-stem diameter measurement to dbh. The form quotient, expressed as a ratio, is always less than or equal to one. A higher ratio means a lower rate of stem taper. Generally, trees grown in the open have a higher rate of taper than trees grown in the forest for a given species. Several types of form quotient have been developed.

In 1899, Schiffel developed the **normal form quotient** which is defined as:

$$q_n = \frac{\text{diameter}_{\text{o.b.}} \text{ at half the tree height}}{\text{dbh}_{\text{o.b.}}}$$

The problem with this measure of form is that if the tree is only 2.6 metres in height, the two diameters are measured at the same point on the tree stem. The differences in form may be due to a difference in tree height rather than in tree form.

The **absolute form quotient** was developed by Jonson in 1910. This measure is defined as:

$$q_a = \frac{\text{diameter}_{\text{o.b.}} \text{ at one - half the height above breast height}}{\text{dbh}_{\text{o.b.}}}$$

The problem with this measure of form is that it varies with a given diameter-height combination within a given species, and, therefore, differences in the absolute form quotient may indicate differences in tree size and form rather than simply differences in tree form.

A more commonly used measure of form is the **Girard form class.** This is defined as:

$$q_G = \frac{\text{diameter}_{\text{i.b.}} \text{ at top of the first 16.0 foot log}}{\text{dbh}_{\text{o.b.}}}$$

The top of the first log was considered to be 16.0 feet plus a 0.3 foot trim allowance plus a 1.0 foot stump. The diameter i.b. was therefore taken at 17.3 feet above ground. This measure was considered to be independent of species except that some allowances were made for excessive butt swell. The advantages to this measure of form are: the top of the first log is close enough to the ground to measure diameter at this point accurately (using telescoping calipers or a relascope); bark thickness is taken into account; and the reference diameter ($\text{dbh}_{\text{o.b.}}$) is near enough to the ground to reflect butt swell.

ESTIMATION OF TOTAL TREE VOLUME

The volume of the main stem, total tree volume, can be estimated from easily measured tree attributes. The most commonly used attributes are dbh, height and form. These estimation equations are called **Volume Equations** or **Volume Functions.**

LOCAL VOLUME FUNCTIONS

Volume functions which are based on a measurement of dbh alone are called **local volume functions.** The assumptions of the local volume equation approach is that for a given diameter of a species (or a group of similar species), the height will be relatively constant. Since this assumption is met only if the trees occupy a small area of land, the term "local" is used. The assumption is not met for large land bases, particularly if there are great changes in elevation and in latitude.

Commonly used local volume equations include:

1. $\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}_i + \varepsilon_i$
2. $\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}_i + \beta_2 \times \text{dbh}_i^2 + \varepsilon_i$
3. $\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}_i^2 + \varepsilon_i$
4. $\text{Volume}_i = \beta_1 \times \text{dbh}_i^{\beta 2} \times \varepsilon_i$

The first equation describes a linear relationship between volume and diameter. Since volume is more closely related to area at breast height (basal area), the relationship between volume and dbh is more likely a curve (i.e., basal area = $\pi \times \text{dbh}^2$). This equation would apply only to a limited range of diameters, as shown in Figure 3.13. In this figure, the equation would apply to dbh values from zero to 40.0 cm.

The second equation is a multiple linear regression equation (more than one independent variable), which describes a parabola (Figure 3.14).

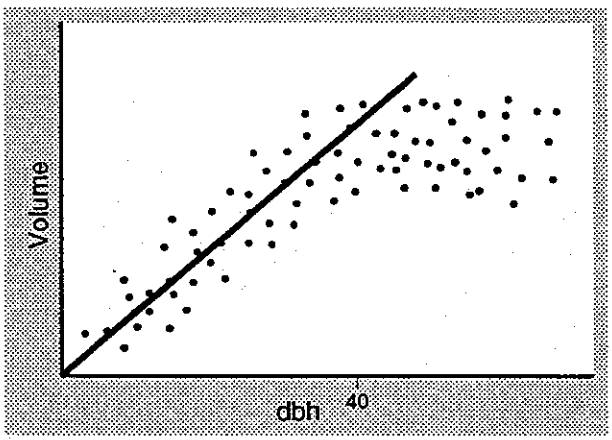


FIGURE 3.13 Linear relationship of volume with dbh for a limited range of DBH values.

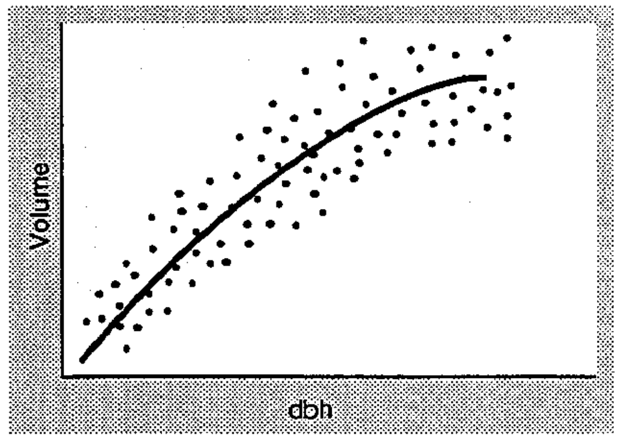


FIGURE 3.14 Parabolic equation for volume with dbh.

This equation more closely resembles the relationship between dbh and volume, until the maximum point when the y value begins to decrease. This would be unreasonable for estimating volume from dbh if we used the function beyond the maximum point of the parabola.

The third function defines a linear relationship between volume and dbh squared. Because volume and basal area are linearly related and dbh squared times π equals basal area, we would expect this function to be a realistic representation of the volume-dbh relationship.

The fourth function is a curvilinear relationship. The function can be transformed using logarithms to be a linear relationship.

$$\log_{10}(\text{Volume}_i) = \log_{10}(\beta_1) + \beta_2 \times \log_{10}(\text{dbh}_i) + \log_{10}(\varepsilon_i)$$

The coefficients for this transformed function can be found using simple linear regression.

Standard Volume Functions

Standard volume functions relate volume to both dbh and height. In Canada, these are the most frequently used total tree volume functions. Because height is included in the equation, it can be used for a larger land area for a given species (or group of related species). Commonly used functions include:

1. $\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}_i^2 \times \text{height}_i + \varepsilon_i$

2. $\text{Volume}_i = \beta_1 \times \text{dbh}_i^2 \times \text{height}_i + \varepsilon_i$

3. $\text{Volume}_i = \beta_1 \times \text{dbh}_i^{\beta 2} \times \text{height}_i^{\beta_3} \times \varepsilon_i$

The first equation describes a linear relationship between volume and dbh squared times height. Since dbh squared times height is similar to the equation used to calculate the volume of the standard shapes (e.g., volume of a cone is π x dbh^2 x height /3), this equation closely describes the relationship between volume and dbh with height. Also, the equation can be fitted using simple linear regression. However, the variance of volume for a given dbh^2 times height varies, so the assumptions of simple linear regression are not met. A weighted least squares regression technique is required to fit this volume function.

The second equation is the same as the first, except that the intercept has been set to zero. This is logical in that when height is zero, dbh^2 times height would also be zero, and thus volume should be zero. When dbh is zero, dbh^2 times height would also be zero, and volume should be close to zero; there may be a small volume because the dbh is at 1.3 metres above ground but volume would be negligible. This function would again be fitted using weighted least squares regression because the variance in volume would likely vary over the range of the *x* variable (dbh^2 times height). Also, the regression would have to be conditioned to have a zero intercept.

The third equation is a curvilinear model. Logarithmic transformations can be used to obtain a linear model.

$$\log_{10}(\text{Volume}_i) = \log_{10} + \beta_2 \times \log_{10}(\text{dbh}_i) + \beta_3 \times \log_{10}(\text{height}_i) + \log_{10}(\varepsilon_i)$$

This model can be fitted using multiple linear regression, unweighted. The unequal variance of volume is usually removed by the logarithmic transformation of volume. Also, the *y* intercept does not appear in the nonlinear equation, so the *y* intercept is zero.

This last equation, the curvilinear standard volume function, is used in B.C. to estimate total tree volume. The equations are fitted from sample data, by species and by region, and then a table of estimated volumes for given dbh and height values is derived using the fitted equations. An example of a B.C. volume table is given as Table 3.3.

From this table, we can find volume for any dbh and height value. For example, for a dbh of 90 cm and a height of 54 m, the estimated tree volume is 10.16 m^3 from the table.

TABLE 3.3 A section of a B.C. Ministry of Forests Volume Table (For mature Douglas-fir over 120 years, Forest Inventory Zones A, B, C.; Logarithmic (base 10) equation for whole stem volume is given as: −4.348375 + 1.692440 log. dbh + 1.181970 log. height.)

D.B.H.	HEIGHT IN METRES														
(cm)	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
10															
15															
20															
25															
30															
35	1.79														
40	2.24	2.41													
45	2.73	2.94	3.14	3.35											
50	3.27	3.51	3.76	4.00											
55	3.84	4.12	4.41	4.70	5.00	5.29	5.59	5.90							
60	4.45	4.78	5.11	5.45	5.79	6.13	6.48	6.83							
65	5.09	5.47	5.85	6.24	6.63	7.02	7.42	7.82							
70	5.77	6.20	6.64	7.08	7.52	7.96	8.41	8.87							
75	6.49	6.97	7.46	7.95	8.45	8.95	9.46	9.97							
80	7.24	7.78	8.32	8.87	9.42	9.98	10.55	11.12	11.69	12.27	12.85	13.44	14.03	14.62	15.22
85	8.02	8.62	9.22	9.83	10.44	11.06	11.69	12.32	12.95	13.59	14.24	14.89	15.54	16.20	16.86
90	8.84	9.49	10.16	10.83	11.50	12.19	12.87	13.57	14.27	14.97	15.68	16.40	17.12	17.85	18.57
95	9.68	10.40	11.13	11.86	12.60	13.35	14.11	14.87	15.64	16.41	17.19	17.97	18.76	19.55	20.35
100	10.56	11.34	12.14	12.94	13.75	14.56	15.39	16.22	17.05	17.90	18.75	19.60	20.46	21.33	22.20
105	11.47	12.32	13.18	14.05	14.93	15.82	16.71	17.61	18.52	19.44	20.36	21.29	22.22	23.16	24.11
110	12.41	13.33	14.26	15.20	16.15	17.11	18.08	19.06	20.04	21.03	22.03	23.03	24.04	25.06	26.09
115	13.38	14.37	15.38	16.39	17.42	18.45	19.49	20.54	21.60	22.67	23.75	24.83	25.92	27.02	28.12
120	14.38	15.45	16.53	17.62	18.72	19.83	20.95	22.08	23.22	24.37	25.52	26.69	27.86	29.04	30.23
125	15.41	16.55	17.71	18.88	20.06	21.25	22.45	23.66	24.88	26.11	27.35	28.60	29.85	31.12	32.39
130	16.46	17.69	18.92	20.17	21.43	22.70	23.99	25.28	26.59	27.90	29.22	30.56	31.90	33.25	34.61
135	17.55	18.85	20.17	21.50	22.85	24.20	25.57	26.95	28.34	29.74	31.15	32.57	34.00	35.44	36.89
140	18.66	20.05	21.45	22.87	24.30	25.74	27.19	28.66	30.14	31.63	33.13	34.64	36.16	37.69	39.24
145	19.81	21.28	22.76	24.27	25.78	27.31	28.86	30.41	31.98	33.56	35.16	36.76	38.38	40.00	41.64
150	20.98	22.53	24.11	25.70	27.31	28.93	30.56	32.21	33.87	35.55	37.23	38.93	40.64	42.36	44.09
155	22.17	23.82	25.48	27.17	28.86	30.58	32.31	34.05	35.80	37.57	39.36	41.14	42.96	44.78	46.61
160	23.40	25.13	26.89	28.67	30.46	32.27	34.09	35.93	37.78	39.65	41.53	43.42	45.33	47.25	49.18
165	24.65	26.48	28.33	30.20	32.09	33.99	35.91	37.85	39.80	41.77	43.75	45.75	47.76	49.78	51.81
170	25.92	27.85	29.80	31.76	33.75	35.75	37.77	39.81	41.86	43.93	46.02	48.12	50.23	52.36	54.50
175	27.23	29.25	31.29	33.36	35.44	37.55	39.67	41.81	43.97	46.14	48.33	50.54	52.76	54.99	57.24
180		30.68	32.82	34.99	37.18	39.38	41.61	43.85	46.12	48.40	50.69	53.00	55.33	57.68	60.03
185		32.13	34.38	36.65	38.94	41.25	43.58	45.93	48.30	50.69	53.10	55.52	57.96	60.41	62.88
190		33.62	35.97	38.34	40.74	43.16	45.60	48.06	50.53	53.03	55.55	58.08	60.63	63.20	65.79
195		35.13	37.58	40.06	42.57	45.10	47.65	50.22	52.81	55.42	58.05	60.69	63.36	66.04	68.74
200		36.67	39.23	41.82	44.43	47.07	49.73	52.41	55.12	57.84	60.59	63.35	66.13	68.93	71.75
205					46.33	49.08	51.85	54.65	57.47	60.31	63.17	66.05	68.96	71.88	74.81
210					48.26	51.12	54.01	56.93	59.86	62.82	65.80	68.80	71.83	74.87	77.93
215					50.22	53.20	56.21	59.24	62.29	65.37	68.48	71.60	74.74	77.91	81.09
220					52.21	55.31	58.44	61.59	64.77	67.97	71.19	74.44	77.71	81.00	84.31
225					54.23	57.45	60.70	63.98	67.28	70.60	73.95	77.33	80.72	84.14	87.58
230					56.29	59.63	63.00	66.40	69.83	73.28	76.76	80.26	83.78	87.33	90.90
235					58.38	61.84	65.34	68.86	72.41	75.99	79.60	83.23	86.89	90.57	94.27
240					60.49	64.08	67.71	71.36	75.04	78.75	82.49	86.25	90.04	93.85	97.69
245					62.64	66.36	70.11	73.89	77.71	81.55	85.42	89.31	93.24	97.18	101.16
250					64.82	68.67	72.55	76.46	80.41	84.38	88.39	92.42	96.48	100.57	104.68

TABLE SHOWS TOTAL VOLUME OF ENTIRE STEM, INSIDE BARK, INCLUDING STUMP AND TOP, WITHOUT ALLOWANCE FOR DEFECT, TRIM OR BREAKAGE
BASED ON 603 TREES STANDARD ERROR OF ESTIMATED VOLUME FOR SINGLE TREES: + OR − 12.3 PER CENT PAGE 2 OF 2

FORM CLASS VOLUME EQUATIONS

Form class volume equations relate volume to dbh, height, and form. A measure of form may be included in the equation, or alternatively, the sample data may be separated by form, and a standard volume equation fitted for each sample set. The fitted equations are then used to create Form Class Volume Tables. Generally, these tables show volume for a given height and dbh class (like the B.C. volume tables), but a different table is given for each Form Class. These Form Class Volume Tables are not often used in Canada, because form is difficult to measure, and often form does not contribute much to the regression fit. Instead, sample data are often separated by species and by land region, to account for differences in tree form, and then standard volume equations are fitted to each set of data.

Photo Volume Equations

Instead of estimating volume from ground measurements, we could also estimate volume from measurements taken on aerial photographs. For example, we may fit the following equation.

$$\text{Volume}_i = \beta_0 + \beta_1 \times \text{height}_i + \beta_2 \times \text{crown width}_i + \varepsilon_i$$

We will discuss how to take measurements on photographs later in the course, and describe how photo volume functions can be obtained using these estimates. Generally, these equations are not as accurate as equations derived solely from ground measurements.

Fitting Volume Equations

A fitted volume equation is needed so that an estimate of volume can be obtained from simple tree measurements. In order to fit the volume equation, measurements of volume and other attributes of interest must be obtained from sample trees. A rule of thumb is that at least 30 sample trees for a given species in a given area are needed to fit a volume equation. Volume can be measured by either using a xylometer, or by sectioning the tree and summing the volume for the sections (see Lesson 2).

As an example of how we can fit a volume function, 12 sample trees were felled, sectioned into short lengths and measured. The dbh and total height, shown in Table 3.4, were recorded for each tree, since our intentions were to fit the following standard volume function:

$$\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}_i^2 \times \text{height}_i + \varepsilon_i$$

The volume of each tree was calculated by summing the volumes of all sections, using Smalian's equation for the middle sections, the cylinder equation for the stump section, and the cone equation for the top section. We can use the information from the sample data to get a fitted equation. First, we will graph the *y* values (volume) versus the *x* values (dbh^2 x height), to see if the assumptions of simple linear regression are met (Figure 3.15).

Table 3.4 Measurements of dbh, height and volume of twelve trees.

Tree No.	dbh (cm)	ht (m)	Vol. (m^3) (y_i)	dbh^2 ht (x_i)	y_i^2	x_i^2	$x_i y_i$
1	27.7	20.28	0.5920	15560.63	0.3505	242133280	9212
2	18.2	13.65	0.2023	4521.42	0.0409	20443248	915
3	31.6	23.10	0.8580	23066.75	0.7362	532074752	19791
4	35.2	28.50	1.1410	35312.63	1.3019	1246981376	40292
5	8.5	10.50	0.0291	758.63	0.0008	575512	22
6	10.1	10.90	0.0479	1111.91	0.0023	1236340	53
7	25.2	18.90	0.3966	12002.25	0.1573	144053904	4760
8	16.2	14.95	0.1409	3923.47	0.0199	15393643	553
9	46.6	31.10	2.3947	67535.50	5.7346	4561043456	161727
10	41.2	29.90	1.8989	50753.43	3.6058	2575910912	96376
11	34.5	23.30	0.9125	27732.83	0.8327	769109504	25306
12	20.1	19.90	0.2671	8039.79	0.0713	64638256	2147
TOTALS			**8.8810**	**250318.94**	**12.8541**	**10173583360**	**361154**

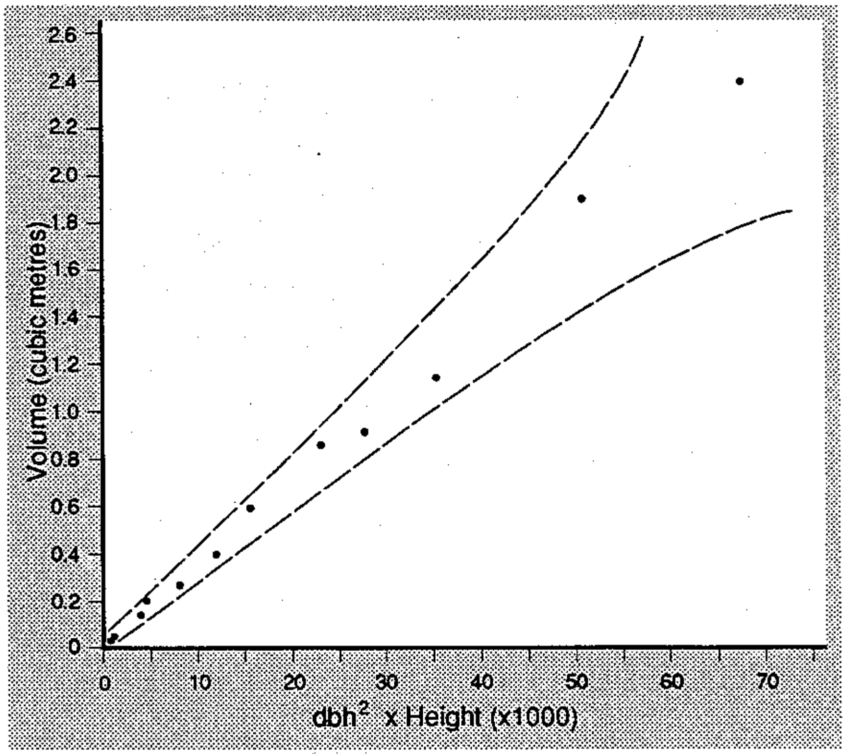


FIGURE 3.15 Graph of volume (*y*) versus dbh^2 times height (*x*).

The relationship between volume (*y*) and dbh^2 times height (*x*) appears to be linear and all other assumptions of simple linear regression appear to be met except that the variance of the *y* values appears to be increasing proportionally to *x* squared. This pattern of variance is common for this standard volume function. Least squares regression can be used to find the estimated regression line as follows:

$$y = \frac{8.8810}{12} = 0.7401; \quad x = \frac{250318.94}{12} = 20859.91$$

$$b_1 = \frac{361154 - \dfrac{250318.94 \times 8.8810}{12}}{10173583360 - \dfrac{(250318.94)^2}{12}} = 0.000035521$$

$$b_0 = 0.7401 - 0.000035521 \times 20859.91 = -0.000865$$

Notice that the values for the *y* variable, volume, are very small relative to the *x* variable, dbh^2 times height. A practical hint for simplifying the calculations for this regression is that we could relate volume to dbh^2 times height divided by 1000 instead of dbh^2 times height and use simple linear regression to fit this alternative equation. In either case, because the variance of the *y* variable is not the same for every *x* value, we cannot calculate confidence intervals in the usual manner. Instead, we could used weighted least squares regression. First, we will transform the equation by dividing by the *x* value (dbh^2 times height).

$$\frac{\text{Volume}_i}{\left(\text{dbh}_i^2 \times \text{height}_i\right)} = \frac{\beta_0}{\left(\text{dbh}_i^2 \times \text{height}_i\right)} + \frac{\beta_1 \times \left(\text{dbh}_i^2 \times \text{height}_i\right)}{\left(\text{dbh}_i^2 \times \text{height}_i\right)} + \frac{\varepsilon_1}{\left(\text{dbh}_i^2 \times \text{height}_i\right)}$$

This simplifies to:

$$y_i = \beta_0 \times \frac{1}{\left(\text{dbh}_i^2 \times \text{height}_i\right)} + \beta_1 + \varepsilon_i'$$

where $y_i = \dfrac{\text{volume}_i}{\left(\text{dbh}_i^2 \times \text{height}_i\right)}$ and ε_i' is the weighted error term.

We can then use simple linear regression to find estimates of β_0 and β_1 for this weighted equation. Notice that β_0 is now a slope (multiplied by 1 over dbh^2 times height) which we will call β_0' and β_1 is now an intercept value for the transformed regression which we will call β_1'. The transformed sample information for the weighted regressions are in Table 3.5. (*Note:* For display purposes, all values in the table have been multiplied by 10^4)

TABLE 3.5 Transformed information for weighted regression taken from a sample of twelve trees.

Tree No.	y_i	x_i	y_i^2	x_i^2	$x_i y_i$
1	0.3804	0.6426	0.00001447	0.00004130	0.00002449
2	0.4474	2.2212	0.00002002	0.00048915	0.00009857
3	0.3720	0.4335	0.00001384	0.00001879	0.00001613
4	0.3231	0.2832	0.00001044	0.00000802	0.00000915
5	0.3836	13.1817	0.00001471	0.01737582	0.00050564
6	0.4308	8.9935	0.00001856	0.00808838	0.00038743
7	0.3304	0.8332	0.00001092	0.00006941	0.00002753
8	0.3591	2.5488	0.00001290	0.00064962	0.00009153
9	0.3546	0.1481	0.00001257	0.00000219	0.00000525
10	0.3741	0.1970	0.00001400	0.00000388	0.00000737
11	0.3290	0.3606	0.00001083	0.00001300	0.00001186
12	0.3322	1.2438	0.00001104	0.00015471	0.00004132
SUM	**4.4169**	**31.0778**	**0.00016429**	**0.02691426**	**0.00122662**

***Note:** All values in the table have been multiplied by 10^4.*

$$\text{SPXY} = \left(0.00122621 \times 10^{-4}\right) - \frac{\left(4.4160 \times 10^{-4}\right)\left(31.0778 \times 10^{-4}\right)}{12} = 8.272487 \times 10^{-9}$$

$$\text{SSX} = \left(0.026914260 \times 10^{-4}\right) - \frac{\left(31.0778 \times 10^{-4}\right)^2}{12} = 1.886567 \times 10^{-6}$$

$$\text{SSY} = \left(0.00016429 \times 10^{-4}\right) - \frac{\left(4.4169 \times 10^{-4}\right)^2}{12} = 1.714953 \times 10^{-10}$$

$$y = \frac{4.4169 \times 10^{-4}}{12} = 3.68075 \times 10^{-5}$$

$$x = \frac{31.0778 \times 10^{-4}}{12} = 2.589816 \times 10^{-4}$$

The estimates of β_0' and β_1' are:

$$b_0' = \frac{8.272487 \times 10^{-9}}{1.886567 \times 10^{-6}} = 4.384942 \times 10^{-3}$$

$$b_1' = \left(3.68075 \times 10^{-5}\right) - \left(4.384942 \times 10^{-3}\right) \times \left(2.589816 \times 10^{-4}\right)$$

$$= 3.567188 \times 10^{-5}$$

The value for β_0' is similar to β_0 from the simple linear regression, and the value for β_1' is similar to β_1 from the simple linear regression, because simple linear regression does give unbiased estimates of the coefficients even if the variances of the y values are not equal.

However, unlike the simple linear regression fit, we can use the results from the weighted regression to test hypothesis statements. For example, to test whether the slope of the original regression is zero (regression is not significant), we use the weighted least square results. First, we need the standard error of the estimate for the weighted regression.

$$\text{SSR} = b_0' \times \text{SPXY} = \left(4.384942 \times 10^{-3}\right)\left(8.272487 \times 10^{-9}\right) = 3.627437 \times 10^{-11}$$

$$\text{SSE} = \text{SSY} - \text{SSR} = 1.714953 \times 10^{-10} - 3.627437 \times 10^{-11} = 1.352209 \times 10^{-10}$$

$$\text{SE}_\text{E} = \sqrt{\text{MSE}} = \sqrt{\frac{\text{SSE}}{(n-2)}} = \sqrt{\frac{1.352209 \times 10^{-10}}{10}} = 3.677239 \times 10^{-6}$$

Since the slope of the original volume equation is now the intercept of the weighted equation, we need to test the intercept of the weighted equation. The standard deviation for the intercept of the weighted equation is as follows:

$$s_{b_1'} = \text{SE}_\text{E}\sqrt{\left(\frac{1}{n}\right)\left(\frac{\sum x_i^2}{\text{SSX}}\right)}$$

$$= 3.677239 \times 10^{-6}\sqrt{\left(\frac{1}{12}\right)\left(\frac{0.02691426 \times 10^{-4}}{1.886567 \times 10^{-6}}\right)}$$

$$= 1.267904 \times 10^{-6}$$

The confidence interval for the intercept of the weighted regression is therefore (for $\alpha = 0.05$):

$$t_{(10,0.975)} = 2.228$$

$$3.567188 \times 10^{-5} \pm (2.228) \times \left(1.267904 \times 10^{-6}\right)$$

$$= \left(3.284698 \times 10^{-5}, 3.849677 \times 10^{-5}\right)$$

Because zero is not in this confidence interval, we conclude that the slope of the original volume equation is not zero, and that the regression is significant.

If the variance of the y values is not proportional to the x values squared, a different transformation is needed than that shown for this volume example. If a different transformation is needed, multiple linear regression is needed to fit the equation.

ESTIMATING MERCHANTABLE VOLUME

Merchantable volume can be estimated directly by changing the "Volume$_i$" in the equations above, with "merch. volume$_i$", and then fitting the equation using the merchantable volume for the sample trees as the y values. However, because we often are interested in an estimate of total volume, and of merchantable volume for several merchantable limits, merchantable tree volume is commonly estimated by reducing the total tree volume by a factor.

We will call the first of these factors the **merchantable ratio**, defined as:

$$MR = \frac{\text{Merchantable Volume}}{\text{Total Volume}}$$

If we have an estimate of the merchantable ratio, and an estimate of the total volume (using one of the functions described in the previous section of this lesson), then we can obtain an estimate of merchantable volume as follows:

$$\text{Est. Merch. Volume} = \text{Est. MR} \times \text{Est. Total Volume}$$

Merchantable ratio is a function of tree size (dbh and height) and also of the merchantable limits. Common functions for estimating merchantable ratio are:

1. $$MR_i = \beta_0 + \beta_1 \times \left(\frac{\text{merch. height}}{\text{total height}}\right)_i + \beta_2 \times \left(\frac{\text{merch. height}}{\text{total height}}\right)_i^2 + \varepsilon_i$$

2. $$MR_i = \beta_0 + \beta_1 \times \left(\frac{\text{d.i.b.}}{\text{dbh}}\right)_i + \beta_2 \times \left(\frac{\text{d.i.b.}}{\text{dbh}}\right)_i^2 + \varepsilon_i$$

3. $$MR_i = \beta_0 + \beta_1 \times \left(\frac{\text{d.i.b.}}{\text{dbh}}\right)_i + \beta_2 \times \left(1 - \frac{\text{merch. height}}{\text{total height}}\right)_i^2 + \varepsilon_i$$

where d.i.b. is the upper merchantable limit (diameter inside bark);
merch. height is the height from ground to the top merchantable limit.

These equations were developed by Honer (1964 and 1967). Other equations include the lower utilization limit (stump height) as an independent variable in the equation.

Another factor used to estimate merchantable tree volume from total tree volume is the reduction factor used in B.C. for quick estimates of merchantable volume.

$$\text{Merch. Volume} = \text{Est. Total Volume} \times (1 - \text{reduction factor})$$

The total volume is found by using the B.C. Total Volume Functions as found in the Volume Tables. The reduction factor is given for the loss in volume for the stump and for the top utilization limits, as a percent, shown in Table 3.6.

TABLE 3.6 Merchantable Factors for Douglas-fir, Forest Inventory Zone C
Constants used to derive whole stem volumes of fir (age range 121 to max years) in cubic metres and factors which are applied to obtain various specified merchantable volumes (note diameter is measured at 1.3 metres).
(From Forest Inventory Division, B.C. Forest Service, Victoria, B.C., 1976.)

Logarithmic (base 10) equation for whole stem volume is: -
Log. Volume = −4.348375 + 1.692440 log. diameter + 1.181970 log. height

Diam. (cm) class	PERCENT REDUCTION TO BE MADE FOR LOSS AT THESE STUMP HEIGHTS		AND TOP DIAMETER LIMITS		
	30 cm	45 cm	10 cm	15 cm	20 cm
10	5.0	6.0	75.0		
15	5.0	6.0	26.0	87.0	
20	5.0	6.0	6.0	45.0	91.0
25	4.0	6.0	3.0	21.0	64.0
30	4.0	5.0	2.0	10.0	38.0
35	4.0	5.0	1.0	5.0	18.0
40	4.0	5.0	.0	3.0	11.0
45	3.0	5.0		3.0	7.0
50	3.0	5.0		2.0	5.0
55	3.0	4.0		1.0	4.0
60	3.0	4.0		1.0	3.0
65	3.0	4.0		.0	2.0
70	3.0	4.0			1.0
75	3.0	4.0			1.0
80	2.0	4.0			1.0
85	2.0	4.0			1.0
90	2.0	3.0			1.0
95	2.0	3.0			1.0
100	2.0	3.0			1.0
105	2.0	3.0			1.0
110	2.0	3.0			1.0
115	2.0	3.0			.0
120	2.0	3.0			
125	2.0	3.0			
130	2.0	3.0			
135	2.0	3.0			
140	2.0	3.0			
145	2.0	3.0			
150	2.0	3.0			
155	2.0	3.0			
160	2.0	3.0			
165	2.0	3.0			
170	2.0	3.0			
175	2.0	3.0			
180	2.0	3.0			
185	2.0	2.0	.0	.0	.0

ESTIMATING NET MERCHANTABLE VOLUME

Net merchantable tree volume is defined as gross merchantable volume less the losses due to unusable wood. To estimate net merchantable volume, gross merchantable volume is first estimated, then reduced to obtain net merchantable volume.

$$\text{Est. Net Merch. Volume} = \text{Est. Merch. Volume} \times (1 - \text{loss ratio})$$

The loss ratio is often expressed as a percent. In B.C., there are three types of percent losses used to obtain net merchantable volume: percent decay, percent waste, and percent breakage (see Lesson 2).

In order to obtain estimates of these losses, sample tree data are needed. Trees are selected, felled, and cut into sections. Each section is measured for diameter and length so that the merchantable volume can be calculated for each tree. In addition, each section is scaled to obtain decay and waste percents for the tree (see Lesson 4 on scaling). For breakage, the amount of volume lost due to breakage is recorded for each tree. Usually, the dbh, height, and presence of damage (conks, scars, broken top, disease and insect damage) are also recorded for each tree. Once sample data showing merchantable volume, and decay, waste, and breakage volumes for each tree are obtained, the estimation of percent decay, percent waste and percent breakage can be performed.

A simple approach to estimating losses is to separate the sample data by species, region, dbh, and risk group, and to calculate a mean percent decay, percent waste, and percent breakage for each of these groups. These values then become the estimated values which can be used to obtain net merchantable volume. Risk group is defined as groups of trees having similar chance of losses. For example, trees with conks are known to have some decay. Trees with scars will likely have more of a chance of decay then trees with no damage recorded.

Another approach to estimating losses is to fit equations to describe the percent loss as a function of tree size, presence of damage, and other tree or stand attributes such as height, site indicators, density, and soil moisture). For example, the following formula could be used to estimate percent decay losses.

$$\text{Percent decay}_i = \beta_0 + \beta_1 \times \text{dbh}_i + \beta_2 \times \text{height}_i + \beta_3 \times \text{risk group}_i + \varepsilon_i$$

In B.C., percent losses have been estimated by separating sample data by species, region, risk group, and maturity class, and then fitting the percent decay, percent waste, and percent breakage each as a function of dbh. The results of these fitted equations have been tabulated. An example of these tables is given in Table 3.7.

TAPER FUNCTIONS

Taper is defined as the change in diameter for the change in height. For a tree, the taper generally appears as in Figure 3.16. This graph is much like the one shown for the volume of solid of revolution in Lesson 2. In fact, in Lesson 2 we said that if the equation which describes the relationship of radius with height from the top of a shape down (in this case, the top of a tree to its stump) were known, we could use integration to obtain the volume of a solid of revolution (i.e., in this case the volume of the tree). Further, we said that we could obtain volume for any subset of the height. This could be used to find the merchantable volume of the tree, or to find the volume of each log of the tree. Also, if we know the height at any point on the tree, we could solve for the radius, and therefore for the diameter at that point. Conversely, if we knew the radius at any point, we would solve for height.

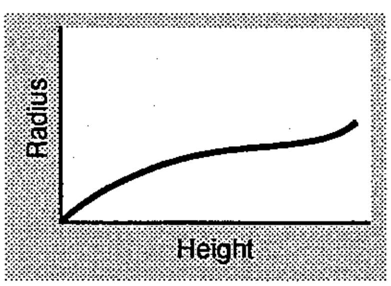


FIGURE 3.16 The relationship between a tree's radius and its height is used to define taper.

Taper functions are used as estimates of the taper curve, and can therefore be used to obtain total and merchantable tree volume, and volume of logs, as well as estimates of diameter or of height at any point on the main stem of the tree. Ideally, one taper function could replace several functions, particularly the total and merchantable tree volume functions.

TABLE 3.7 B.C. Ministry of Forests Table of Decay Waste and Breakage Percentage Reduction Factors Used in Volume Computation of Standing Samples, for Douglas-fir, Age Range 121 to 251+

Diam.	GROUP 1			GROUP 2			GROUP 3		
Class	Decay	Waste	Breakage	Decay	Waste	Breakage	Decay	Waste	Breakage
10	.4			.4			50.0		
15	1.2	1.2	5.0	1.5	.0	5.0	50.6	17.0	5.0
20	2.0	1.5	5.0	2.6		5.0	49.7	17.2	5.0
25	2.7	2.5	5.0	3.5	.0	5.0	48.5	17.7	5.0
30	3.3	3.2	5.0	4.1	.3	5.0	47.5	17.9	5.0
35	3.7	3.3	5.0	4.5	.6	5.0	46.3	18.3	5.0
40	4.0	3.2	5.0	4.7	1.1	5.0	45.3	18.3	5.0
45	4.1	2.8	5.0	4.7	1.4	5.0	44.3	18.3	5.0
50	4.0	2.4	5.0	4.6	1.5	5.0	43.2	18.3	5.0
55	3.8	1.8	5.0	4.2	1.5	5.0	42.0	18.5	5.0
60	3.6	1.0	5.0	3.8	1.4	5.0	41.0	18.2	5.0
65	3.2	.2	5.0	3.5	1.3	5.0	40.0	17.9	5.0
70	2.7	.1	5.0	3.3	1.2	5.0	38.8	17.7	5.0
75	2.4	.0	5.0	3.2	1.2	5.0	37.7	17.5	5.0
80	1.9		5.0	3.2	1.2	5.0	36.7	17.1	5.0
85	1.6		5.0				35.6	16.9	5.0
90	1.4		5.0				34.6	16.5	5.0
95	1.2		5.0				33.6	16.2	5.0
100	1.0		5.0				32.5	15.8	5.0
105							31.4	15.6	5.0
110							30.3	15.4	5.0
115							29.3	14.9	5.0
120							28.2	14.7	5.0
125							27.2	14.2	5.0
130							26.1	13.9	5.0
135							25.0	13.5	5.0

GROUP 1: Any living tree which (a) has no indicators, or (b) has indicators in only one of these categories: dead and/or broken tops, large rotten branches, or frost cracks.
GROUP 2: Any living tree which (a) has indicators in only one of these categories: mistletoe, forks and/or crooks, or scars, or (b) has any combination of indicator categories, but has no conks or blind conks.
GROUP 3: Any living tree having conks or blind conks.

The problem in finding a taper function is that the equation describing tree shape is much more complex than for standard shapes, such as a cone or paraboloid. In fact, we really need three functions: one for the base of the tree, one for the middle, and one for the top. The points where the shapes change are called **inflection points.** We have several options in fitting taper functions.

1. We can ignore the differences in shapes and use one simple equation to describe the tree. The result would be that the volumes from this simple taper function would not be very accurate.

2. We can include the variation at the stump by using a more complex equation, or by fitting two separate equations and conditioning them to join at the lower inflection point. This would be more accurate than alternative 1, but more difficult, particularly since the point on the tree where the shape changes, the lower inflection point, is difficult to define.

3. We could include the three shapes by fitting three equations and conditioning them to join at the two inflection points, or by using a detailed equation. This would be the most accurate method.

The problem with more complicated taper functions is that they often cannot be integrated. The volume must be found by estimating the diameter at many heights along the tree stem, calculating volume for these small disks and summing the disk volumes to obtain tree volume (this process is called **iterations**). This requires much computer time, and some of the advantages of the taper function over the total and merchantable tree volume functions are lost. As computers increase in power and speed, however, this problem with complex taper functions becomes less of a concern.

Below is an example of a simple taper function (by Kozak, Munro & Smith, 1969):

$$\frac{d_i^2}{\mathrm{dbh}^2} = \beta_1 \times \left(\frac{h_i}{h} - 1\right) + \beta_2 \times \left(\frac{h_i^2}{h^2} - 1\right) + \varepsilon_i$$

where d_i is the diameter inside bark at the point h_i from the top of the tree; h is the total tree height.

Instead of describing the taper by the radius at a point on the tree stem, this function describes the taper by the diameter at a point on the tree stem. The two estimated slope values, b_1 and b_2, can be found using multiple linear regression. Once the coefficients are estimated, we could calculate volume for the entire tree by using calculus as in Lesson 2. First, we will rewrite the taper function:

$$d_i^2 = \mathrm{dbh}^2 \times b_1 \times \left(\frac{h_i}{h} - 1\right) + b_2 \times \left(\frac{h_i^2}{h^2} - 1\right)$$

We can then define the calculation of area at any point on the tree stem by:

$$\text{Area} = \pi \times \frac{d_i^2}{4} = \frac{\pi}{4} \times \mathrm{dbh}^2 \times \left(b_1 \times \left(\frac{h_i}{h} - 1\right) + b_2 \times \left(\frac{h_i^2}{h^2} - 1\right)\right)$$

The volume of the entire tree, from the ground where h_i is zero, to the tip where h_i is equal to the tree height, is then found by: (*Note:* h_i replaced with x for the integration)

$$\begin{aligned}
\text{Volume} &= \int_0^h \text{area } dx \\
&= \int_0^h \frac{\pi}{4} \times \mathrm{dbh}^2 \times \left(b_1 \times \left(\frac{x}{h} - 1\right) + b_2 \times \left(\frac{x^2}{h^2} - 1\right)\right) dx \\
&= \frac{\pi}{4} \times \mathrm{dbh}^2 \int_0^h \left(b_1 \times \frac{x}{h} - b_1 + b_2 \times \frac{x^2}{h^2} - b_2\right) dx \\
&= \frac{\pi}{4} \times \mathrm{dbh}^2 \left. \left| \, b_1 \times \frac{x^2}{2h} - b_1 \times x + b_2 \times \frac{x^3}{3h^2} - b_2 \times x \right. \right|_0^h \\
&= \frac{\pi}{4} \times \mathrm{dbh}^2 \times \left(b_1 \times \frac{h^2}{2h} - b_1 \times h + b_2 \times \frac{h^3}{3h^2} - b_2 \times h\right) \\
&= \frac{\pi}{4} \times \mathrm{dbh}^2 \times \left(b_1 \times \frac{h}{2} - b_1 \times h + b_2 \times \frac{h}{3} - b_2 \times h\right) \\
&= \frac{\pi}{4} \times \mathrm{dbh}^2 \times h \times \left(b_1 \times \frac{-1}{2} + b_2 \times \frac{-2}{3}\right)
\end{aligned}$$

The final equation can be used to find volume by entering the dbh, the total height, and the estimated slopes, b_1 and b_2. The fitted taper equation can also be written to calculate height for a given diameter of the tree stem, and to calculate merchantable volume. The fit of a single taper function therefore results in equations for diameter at a height, height at a diameter, total volume, and merchantable volume.

REVIEW/SELF-STUDY QUESTIONS

Do these questions before you go on to complete the Graded Assignment. These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. What is the main difference between simple and multiple linear regression?
2. Why is simple linear regression also called "least squares" regression?
3. What are the six assumptions which must be met in order to use simple linear regression? What are the effects if each of these assumptions is not met?
4. Define coefficient of determination and standard error of the estimate.
5. Why do we calculate confidence intervals for the slope and for the intercept?
6. Describe two ways to test whether the simple linear regression is significant. If a regression is significant, what does this mean?
7. Differentiate between form factor and form quotients.
8. What is the difference between a local volume function and a standard volume function? When should a local volume function be used?
9. What independent variables are commonly used to estimate the merchantable ratio and why?
10. How is net merchantable volume currently estimated in B.C.?
11. Describe the advantages and disadvantages of using a taper function to estimate total and merchantable volume.

Lesson 4 Log Scaling

Introduction

Lesson Overview

This lesson presents approaches to determining the volume and quality of logs. Scaling is important because the volume and quality of logs harvested ultimately determine the value of the stand which in turn influences the return (stumpage) to the landowner (the Province in the case of public land). Scaling is also an important component in assessing whether usable wood has been left on the ground after harvesting has been completed (residue surveys). Scaling systems are specific to a particular jurisdiction (e.g., British Columbia). For this reason, this lesson contains much information specific to British Columbia's scaling procedures. However, many of the principles remain the same in other parts of the world.

Lesson Objectives

After completing this lesson and the assignment, you should be able:

1. to distinguish between the major scaling methods;
2. to scale a log using the B.C. Metric Scale;
3. to state the major factors that influence log grade;
4. to describe examples of each log grade, including in the examples each major factor contributing to the grade.

Lesson Readings

Avery and Burkhart (pages 53–95) provide an American perspective on the topic of log scaling. If you are interested in more detail on scaling in British Columbia, the Ministry of Forest's *Scaling Manual* or the *Forestry Handbook for British Columbia* may be borrowed from the Extension Library; the latter book may be purchased from the UBC Bookstore. (See page vi for information about obtaining items from the Extension Library.) We recommend you complete this lesson before you read these books.

Lesson Assignment

Answer the self-study questions at the end of this lesson before you complete Graded Assignment #3 in Appendix A. Mail the Graded Assignment to your tutor by the date indicated on your course schedule. Don't forget to include a pink assignment cover sheet.

SCALING

Scaling involves the measurement of log volumes. A component of scaling, **log grading**, consists of assigning logs to particular quality classes. You will see as you work your way through this lesson that "art" as well as "science" are involved in the techniques of scaling and grading. For this reason, the scaling and grading procedures that have been developed in various jurisdictions (e.g., the province of British Columbia) differ. These procedures usually are based on legislation specific to the jurisdiction. Individuals generally must be licensed in order to carry out the procedures officially within a jurisdiction. Such individuals are called **scalers.**

Identification of the species and origin of logs are important components of scaling. Species identification is generally made from bark and wood characteristics. The origin of logs is generally indicated by a stamped symbol called a **timber mark.** We will not discuss these aspects of scaling any further, although scalers are required to have a detailed comprehension of them.

In this lesson, we initially present some background on scaling methods in general. We follow this with a detailed summary of stick and weight scaling procedures practiced in British Columbia. Finally we provide a brief overview of the provincial grading procedures.

Why Logs are Measured for Quality and Quantity

Timber is scaled and graded in order to determine its value. Land owners usually receive payment (called **stumpage**) from other individuals or companies for the right to harvest their timber. Stumpage systems often vary among jurisdictions, and may vary among contractual agreements (**tenure types**) within a jurisdiction. Almost all stumpage systems depend in part on the value of the stand. Scaling and grading of logs after harvesting, and/or estimating log scale and grade from standing trees, is an important component of these stumpage systems. Log value is also used for buying and selling logs, and sometimes for payment of logging contractors.

Values are assigned to logs based upon measures of quantity and quality. Quality depends on criteria which affect the end use of the log. Some of these criteria are described in the latter part of this lesson. Quantity can be measured in a number of ways. The principle methods used over the years have included the number of logs, their weight, and their volume. Number of logs, a simple measurement, is not frequently used by itself because it does not reflect log size which often has a major influence on value of a log. Weight and volume are directly related to log size. It is easier to measure weight than volume, but the relationship between weight and value can be confounded by the inclusion of bark, mud, and ice, as well as varying moisture content of the wood. Hence, volume is regarded as the most precise method for obtaining an indication of quantity in scaling. In situtations where weight scaling is used, the weights are frequently related to volume and quality through sampling. Samples should be representative of specific species, for a given region and season. We discuss how this procedure is applied in British Columbia later in this lesson.

volume units

Historically, scaled volume was most often recorded in units pertaining to the common end product of the log—lumber. Volume of lumber is usually recorded in a nominal unit known as a **board foot**, which represents a volume of 12 inches by 12 inches by 1 inch. Standardized procedures known as **board foot log rules** were developed to relate measures of log length and diameters to a volume in board feet. Numerous log rules were developed over the years, each based on different assumptions of board sizes, saw widths **(kerf)** and board length. Either diagrams or mathematical formulae were used in the derivation. It is extremely rare that actual lumber output from a run of logs exactly matches the predicted board foot scale, although some log rules are more accurate than others. We do not discuss log rules in any more depth in this course.

The recent trend in log scaling in Canada is to use cubic volume units (m^3). These units have become popular principally for two reasons: the units are standardized and not dependent upon assumed sawing standards; and the units are appropriate for other timber products (e.g., pulp and firewood). If you are interested in obtaining additional background about log rules, see Avery and Burkhart (pages 45–52).

Calculating the volume of logs in cubic units has been covered in Lesson 2. For scaling purposes, it is essential that the measurements required to obtain volume for a given log be made quickly since often many logs are scaled at the same time. This means that one of the sectional volume formulae you learned in Lesson 2 (Smalian's, Huber's, or Newton's formula) is generally used to convert measurements of length and diameter inside bark to volume. Of these three formulae, Smalian's is used most frequently since it requires only log length and diameter inside bark at the ends of logs. Huber's and Newton's formulae both require diameter inside bark at the midpoint of the log. This is difficult to measure if bark is present or if the logs are located in the interior of a pile.

"STICK" SCALING IN BRITISH COLUMBIA

THE SCALING STICK

Logs frequently are measured for diameter (or radius) using a **scaling stick**. Scaling sticks do not differ much in basic design. However, numbers printed on the face of the stick pertain to a specific log rule and differ among jurisdictions. A drawing of the scaling stick used in British Columbia is given in Figure 4-1. You will get a chance to work with a scaling stick during your on-campus laboratory session. The main body is comprised of fibreglassed wood. The tine on one end of the scaling stick is metal and the handle is cork. Common lengths are one, one and a half, and two metres, not including the handle.

Three distinct sets of numbers are printed on the main body of the scaling stick. These numbers are used to measure inside bark radius and, occasionally, length, reducing length or radius measurements to reflect the amount of decay present in the log, and calculating log volume. In order to understand what these numbers represent, it is necessary to become familiar with British Columbia's scaling system. We introduce various elements of the system as this lesson proceeds.

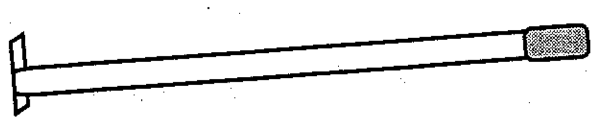


FIGURE 4.1 Drawing of a scaling stick. Numbers on the scaling stick face are reproduced in Figure 4.3.

Measuring Log Radius

Log radius inside bark at both ends of a log is measured using the numbers printed on the edge of the scaling stick. These numbers are divided into 2 cm classes, numbered consecutively from 1 up to the maximum possible for the length of stick. The scaler places the metal tine inside the bark at one side of the log and lays the stick so that it passes by the log pith to the opposite side of the log. A record is made of the class number in which the opposite edge of the log (inside the bark) falls. Essentially this procedure consists of the scaler measuring the diameter inside bark, but recording the radius. (Since the radius is one-half of the diameter, labelling 2 cm classes in increments of 1 automatically converts the diameter measured to 1 cm radius classes.) This is a much easier way for the scaler to measure average radius than trying to measure it directly because the pith seldom falls in the exact centre of a log. If the log is not circular in cross-section, two measurements are made at right angles to one another, and the average is recorded.

Radius is recorded to the nearest centimetre. On the scaling stick this means that any reading which falls between the lines demarking a radial class is recorded as that particular class. If the reading falls directly onto a line dividing classes, the reading is rounded to the nearest even-numbered class. For example, if a reading falls directly on the line dividing the 15 and 16 cm classes, 16 cm would be recorded; if the reading were to fall directly on the line dividing the 16 and 17 cm classes, 16 cm would again be recorded.

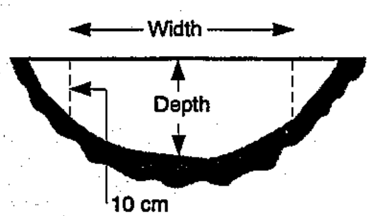


Figure 4.2 Measurements taken on a log slab.

Some special situations that may be encountered when measuring radius are **butt flare** and **slabs.** Butt flare (butt swell) is sometimes present in the butt logs of certain trees. Some species (e.g., western redcedar) often have this flare. This flare should not be included as part of the radius measurement at the bottom end of a butt log because it would result in an overestimate of the volume. The scaler should either measure the radius above the flare or estimate how much the flare adds to the radius, and subtract that amount before recording the radius.

Slabs are occasionally found in a log pile. Radii are recorded for slabs as if they were round logs. The scaler measures the slab width (defined as the distance between points where 10 cm faces could be obtained) and depth (defined as the maximum thickness of the slab) at each end of the log (see Figure 4.2). Each radius is then estimated from:

$$\text{Radius Recorded} = \frac{\text{Width} + \text{Depth}}{4}$$

Measuring Log Length

The length of logs can be measured using either the scaling stick or a **logger's tape.** A logger's tape is a metal measuring tape wound onto an open-edged metal spool. There are claws on the zero end of the tape for attaching the tape to one end of the log. The scaler needs simply to stretch the tape along the length of the log to determine the length. If the scaling stick is used, it is laid repeatedly along the log until the complete log length is covered. The scaler uses the heel of the metal tine to mark the end of each stick length. The scale along the narrow edge of the scaling stick used for measuring radius is used to measure the length remaining (fractional length) when another complete stick length would extend past the end of the log. The class number on the stick edge will give the fractional length in cm if it is multiplied by 2. (Recall that the class widths are 2 cm.) The length of the log is then determined as:

Number of Stick Lengths x Length of Stick + Fractional Length

Of the two methods of measuring length, the logger's tape is preferred because it is less likely to produce errors.

Log lengths are always recorded in m to the nearest 0.2 m. Rounding is always towards the nearest 0.2 m division. For example, a length of 8.09 m would be recorded as 8.0 m; a length of 8.11 m would be recorded as 8.2 m. Borderline lengths are always rounded down. For example, a length of 8.10 m would be recorded as 8.0 m; a length of 8.3 m would be recorded as 8.2 m.

CALCULATING VOLUME

using Smalian's formula

In British Columbia, scaled volumes are determined in cubic meters (m^3) and recorded to three decimal places. **Smalian's formula** is used to calculate volume from measurements of log radius at the top and bottom ends of the log, and log length.

Let's look at the volume determination for a log of length 8.0 m, with a top radius inside bark of 8 cm and a bottom radius inside bark of 16 cm. In order to simplify the presentation of log dimensions, we will adopt a convention of writing the dimensions as $L/R_T/R_B$ with no units. This log can then be described as 8.0/8/16. We will first calculate the volume using Smalian's formula directly and then show you how to compute the same volume using some of the numbers on the face of the scaling stick.

Recall from Lesson 2 that Smalian's formula can be written as:

$$V_l = \frac{\left(\pi R_T^2 + \pi R_B^2\right)}{2} \times L$$

where V_l is the volume of the log in m^3, R_T and R_B are the radii at the top and bottom of the log respectively in metres, and L is the length of the log in metres. Using this formula, we can compute the volume for our log as:

$$V_l = \frac{\left(\pi \times 0.08^2 + \pi \times 0.16^2\right)}{2} \times 8.0 = 0.402 \text{ m}^3$$

using the scaling stick for lengths displayed on the stick

The **scaling stick** can be used to aid in calculating log volumes without the need for a calculator. In order to follow the steps which we will go through, you should refer to the the scaling stick template displayed in Figure 4.3 on the following page.

Step 1. Go to the column with radius 8.

Step 2. Go up the column until you reach the number at the same level as the 8 m length (recorded in the first column).

Step 3. Write that number down. (80)

Step 4. Go to the column with radius 16.

Step 5. Go up the column until you reach the number at the same level as the 8 m length.

Step 6. Write that number down. (322)
Step 7. Add these numbers together. (80 + 322 = 402)
Step 8. Divide the sum by 1000. (402 ÷ 1000 = 0.402)

The resulting number is the volume of the log in cubic metres.

In order to show you why this works, it is necessary to first rewrite Smalian's formula.

$$V_1 = \frac{\left(\pi R_T^2 + \pi R_B^2\right)}{2} \times L = \frac{\left(A_T + A_B\right)}{2} \times L$$
$$= \frac{A_T \times L}{2} + \frac{A_B \times L}{2}$$

A_T and A_B are the cross-sectional areas in square metres at the top and bottom ends of the log respectively. What this version of Smalian's formula is saying in words is that the volume of a log is the sum of the half volume of a cylinder of length L formed using the cross-sectional area at the top of the log ($A_T \times L/2$)

METRIC SCALE STICK

Length (m) \ Radius (cm)	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
12 m	30	47	68	92	121	153	188	228	271	319	369	424	483	545	611	680
11 m	28	43	62	85	111	140	173	209	249	292	339	389	442	499	560	624
10 m	25	39	57	77	101	127	157	190	226	265	308	353	402	454	509	567
9 m	23	35	51	69	90	115	141	171	204	239	277	318	362	409	458	510
8 m	20	31	45	62	80	102	126	152	181	212	246	283	322	363	407	454
Factor	5	8	11	15	20	25	31	38	45	53	62	71	80	91	102	113
7 m	18	27	40	54	70	89	110	133	158	186	216	247	281	318	356	397
6 m	15	24	34	46	60	76	94	114	136	159	185	212	241	272	305	340
5 m	13	20	28	38	50	64	79	95	113	133	154	177	201	227	254	284
4 m	10	16	23	31	40	51	63	76	90	106	123	141	161	182	204	227
3 m	8	12	17	23	30	38	47	57	68	80	92	106	121	136	153	170

Factor = full volume of 1 m log in dm³ = running metre volume in dm³

Half volume of logs in dm³

Length (m) \ Radius (cm)	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
12 m	754	831	912	997	1086	1178	1274	1374	1478	1585	1696	1811	1930	2053	2179	2309
11 m	691	762	836	914	995	1080	1168	1260	1355	1453	1555	1660	1769	1882	1997	2117
10 m	628	693	760	831	905	982	1062	1145	1232	1321	1414	1510	1608	1711	1816	1924
9 m	565	623	684	748	814	884	956	1031	1108	1189	1272	1359	1448	1540	1634	1732
8 m	503	554	608	665	724	785	849	916	985	1057	1131	1208	1287	1368	1453	1539
Factor	126	139	152	166	181	196	212	229	246	264	283	302	322	342	363	385
7 m	440	485	532	582	633	687	743	802	862	925	990	1057	1126	1197	1271	1347
6 m	377	416	456	499	543	589	637	687	739	793	848	906	965	1026	1090	1155
5 m	314	346	380	415	452	491	531	573	616	661	707	755	804	855	908	962
4 m	251	277	304	332	362	393	425	458	493	528	565	604	643	684	726	770
3 m	188	208	228	249	271	295	319	344	369	396	424	453	483	513	545	577

Length (m) \ Radius (cm)	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
12 m	2443	2581	2722	2867	3016	3169	3325	3485	3649	3817	3989	4164	4343	4526	4712
11 m	2239	2365	2495	2628	2765	2905	3048	3195	3345	3499	3656	3817	3981	4149	4320
10 m	2036	2150	2268	2389	2513	2641	2771	2904	3041	3181	3324	3470	3619	3771	3927
9 m	1832	1935	2041	2150	2262	2376	2494	2614	2737	2863	2991	3123	3257	3394	3534
8 m	1629	1720	1815	1911	2011	2112	2217	2324	2433	2545	2659	2776	2895	3017	3142
Factor	407	430	454	478	503	528	554	581	608	636	665	694	724	754	785
7 m	1425	1505	1588	1672	1759	1848	1940	2033	2129	2227	2327	2429	2533	2640	2749
6 m	1221	1290	1361	1434	1508	1584	1663	1743	1825	1909	1994	2082	2171	2263	2356
5 m	1018	1075	1134	1195	1257	1320	1385	1452	1521	1590	1662	1735	1810	1886	1963
4 m	814	860	907	956	1005	1056	1108	1162	1216	1272	1330	1388	1448	1509	1571
3 m	611	645	680	717	754	792	831	871	912	954	997	1041	1086	1131	1178

FIGURE 4.3 British Columbia scaling stick template.

with the half volume of a cylinder of length L formed using the cross-sectional area at the bottom of the log ($A_B \times L/2$). When we calculate this for our example, we get:

$$V_1 = \frac{\pi R_T^2 \times L}{2} + \frac{\pi R_B^2 \times L}{2} = \frac{\pi \times 0.08^2 \times 8.0}{2} + \frac{\pi \times 0.16^2 \times 8.0}{2}$$
$$= 0.80 + 0.322$$

Note that these numbers correspond to the numbers we found on the scaling stick except that they are divided by 1000.

We can now define the numbers that we looked up on the scaling stick. These numbers represent the half volume of a cylinder of a particular length and radius recorded in dm^3 (cubic decimetres). There are 10 dm in 1 m and 1000 dm^3 in 1 m^3. Hence we need to divide the numbers on the scaling stick by 1000 to get a volume in m^3. The reason that the half volumes are recorded on the scaling stick in these units is to avoid the need for a decimal place.

That is all that there is to calculating volumes using the scaling stick providing that the length of the log for which you wish to calculate volume is given on the stick (i.e., log lengths between 3 m and 12 m inclusive in intervals of 1 m). Before moving on to learn about how to handle other lengths, you may wish to calculate a few log volumes on your own by using the scaling stick template. The dimensions of a few practice logs are given below.

Log 1	12.0/25/35	*(Answer: 3.487 m^3)*
Log 2	9.0/ 6/21	*(Answer: 0.674 m^3)*
Log 3	8.0/10/20	*(Answer: 0.629 m^3)*

using the scaling stick for lengths that are not displayed

Smalian's formula can be considered as the average of two cylinders; one with a radius R_T, the other with radius R_B, and both having a length of L. Since the volume of a cylinder is directly proportional to length, you can mathematically separate the log into parts with different lengths but identical radii, calculate the volume of each part, and sum the results to obtain the overall volume. This can be illustrated symbolically as:

$$V_1 = \frac{\left(\pi R_T^2 + \pi R_B^2\right)}{2} \times L$$
$$= \frac{\left(\pi R_T^2 + \pi R_B^2\right)}{2} \times L_1 + \frac{\left(\pi R_T^2 + \pi R_B^2\right)}{2} \times L_2$$
$$= \frac{\pi R_T^2 L_1}{2} + \frac{\pi R_B^2 L_1}{2} + \frac{\pi R_T^2 L_2}{2} + \frac{\pi R_B^2 L_2}{2}$$

where $L_1 + L_2 = L$.

As can be seen from above, you would need to sum four readings from the scaling stick to determine the volume of a log separated into two parts. In theory, a log can be separated into as many parts as you would like. The number of scaling stick readings you would need to sum to obtain the log volume is always twice the number of parts. In practice, it is most efficient to keep the number of parts to a minimum.

As an example, let's look at a log with dimensions 17.0/14/36. A length of 17 m is not found on the scaling stick, so if we wish to use the stick to calculate volume we must break 17.0 m into parts. Say we choose to break the log into 12.0 m and 5.0 m parts. The scaling stick readings are:

12.0/14	369	
12.0/36	2443	
5.0/14	154	
5.0/36	1018	
	3984	÷ 1000 = 3.984 m^3

Does it make any difference if we choose other segment lengths? Based on the mathematics presented previously, hopefully you see that the answer remains the same. This can be illustrated quite easily. Say we chose component lengths of 10.0 m and 7.0 m. The scaling stick readings would be:

10.0/14	308	
10.0/36	2036	
7.0/14	216	
7.0/36	1425	
	3985	÷ 1000 = 3.985 m^3

The small difference of 0.001 m^3 is due only to rounding.

Half volumes within any column on the scaling stick can be added together to produce the half volume of a cylinder having a length equal to the sum the component lengths. Multiplying or dividing numbers in a column to produce half volumes for other lengths also works. For example, the reading for 4.0/4 is 10. If this is multiplied by 2 it yields the reading for 8.0/4 which is 20.

Proportionality does not hold between columns because each column represents a different radius; volume is proportional to the square of the radius, not radius itself. This can be easily demonstrated. The half volume associated with 3.0/9 is 38 dm^3 while the half volume associated with an identical length but triple the radius (3.0/27) is 344 dm^3. Dividing 344 by 38 yields 9 (3^2). If a proportional relationship did exist between columns, the answer would have been 3.

Let's calculate the volume of a log with dimensions 7.4/7/9 using the scaling stick. We can divide the length into parts of 7.0 m and 0.4 m. To find the appropriate readings for 0.4 m, we simply have to find the readings for a length of 4.0 m and divide them by 10. Hence, the volume of the log can be calculated as:

0.4/7	3	(4.0/7 ÷ 10)
0.4/9	5	(4.0/9 ÷ 10)
7.0/7	54	
7.0/9	89	
	151	÷ 1000 = 0.151 m^3

As another example, let's consider a log with dimensions 15.0/35/45. We could calculate the volume using the scaling stick by separating 15.0 m into two components, say 10.0 m and 5.0 m. If we take this approach we would have to look up four values. A simpler approach would be to look up the values for 5.0/35 and 5.0/45 and multiply them by 3.

15.0/35	2886	(5.0/35 x 3)
15.0/45	<u>4770</u>	(5.0/45 x 3)
	7656	÷ 1000 = 7.656 m^3.

THE SCALING STICK FACTOR

If you look at your scaling stick template (Figure 4.3), you will see a series of numbers at right angles to the other numbers. These numbers are called **factors,** and there is one associated with each radius class. The factor is the volume in dm^3 of a 1 m long cylinder of that radius.

As an example, let's calculate the factor for a radius of 15 cm. Recall that the volume of a cylinder is $\pi R^2 \times L$. Since we wish the volume to be in dm^3 the radius and length must be expressed in dm (i.e., R = 1.5 dm and L = 10 dm). Therefore, the factor for a radius of 15 equals $\pi \times 1.5^2 \times 10$ which equals 71 dm^3. You can verify that this number is identical to that printed on the scaling sheet template. The factor for any other radius class can be calculated in a similar manner.

Factors can be used to determine quickly the areas of circles of various radii. To obtain an area in m^2 it is necessary to divide the factor by 1000. To obtain an area in dm^2, the factor needs to be divided by 10. We will illustrate this using a radius of 15 cm again. The area of a circle is πR^2. If the area is to be in m^2, then we would use .15 m as the radius. The area would be $\pi \times .15^2$ which equals 0.071 m^2 (i.e., 71 ÷ 1000). If the area is to be in dm^2, then we would use 1.5 dm as the radius. The area would be $\pi \times 1.5^2$ which equals 7.1 dm^2 (i.e., 71 ÷ 10).

The principle use of the factor is for reducing the dimensions of logs to account for the volume of decay present. This is the subject we deal with next.

REDUCING LOG DIMENSIONS TO ACCOUNT FOR DECAY

Up to this point in this lesson we have talked about the easy part of scaling. The scaled volume that we want to obtain is the net volume of the log inside bark (i.e., the amount of solid wood present). Therefore, we must somehow take the amount of decay into account.

The basic premise on which the scaler operates is that only three measurements are recorded: length of the log, radius at the top end of the log, and radius at the bottom end of the log. This is fine if there is no rot present, but if there is rot (or other defects which detract from the usable solid wood content of the log) what can the scaler do? In theory, all the scaler has to do is to reduce one or more of the measured dimensions to provide a new set of dimensions which reflect the true firmwood volume present. How the scaler does this depends on the type of defect present.

A major portion of the art of scaling comes into recognizing the type and amount of defect present. In order for you to learn this effectively you would have to spend a considerable amount of time scaling actual logs with a knowledgeable scaler. A course in scaling sponsored by the Ministry of Forests is available periodically at various locations throughout the Province. Successful completion will provide you with a Scaler's Licence. It is not our intention in this lesson to duplicate that kind of practice. Instead, we will examine some of the more common types of rot, and for each type of rot, we will illustrate the principles behind the kind(s) of reduction which can be made.

sap rot

Sap rot is rot around the outside portion of the log (Figure 4.4). The volume of firmwood when sap rot is present is simply the volume of the solid central core of the log. If the rot is present at both ends of the log, a reduction is made to each of the radii to reflect the radii of the solid wood core. This is done by measuring the radii inside the rot.

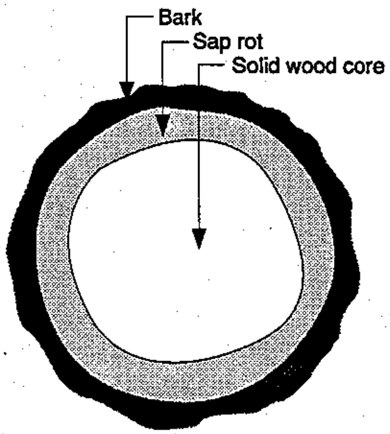


Figure 4.4 End view of sap rot in a log.

As an example, consider a log with total dimensions of 10.0/20/27 having 2 cm of sap rot at the top and 4 cm at the bottom end. The gross volume (volume without deducting the rot) of that log is 1.773 m^3. The dimensions that the scaler would record (the revised dimensions) are 10.0/18/23. This yields a firmwood volume of 1.340 m^3.

If sap rot is visible only at one end of the log, then the radius inside the rot is recorded at the end at which the rot is present. The total radius is recorded at the opposite end. Implicit in this reduction is the assumption that the rot extends halfway through the log as a cylinder or tapers off exactly at the top end of the log.

Unlike the other kinds of rot we will describe, reducing the dimensions for sap rot always yields an exact firmwood volume. In other words, the firmwood volume we can calculate from the original log and rot dimensions is exactly the same as that yielded by the reduced dimensions.

heart rot that extends through the log

Heart rot is in a sense the opposite of sap rot. The decay is present as a central core in the log and is surrounded by a solid shell of wood (Figure 4.5). Since the methods for reducing log dimensions differ depending upon whether the rot extends through the log (i.e., visible at both ends) or not, we consider the two cases separately.

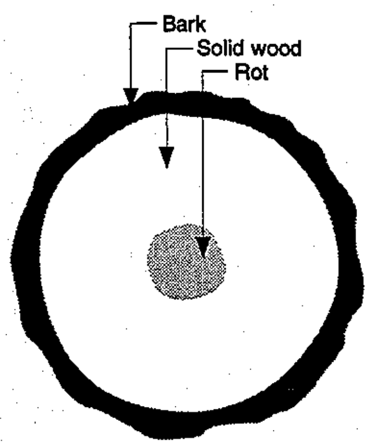


Figure 4.5 End view of heart rot in a log.

We will illustrate the various procedures appropriate in the case of heart rot that extends through the log by means of an example. The log we use as the example has total dimensions of 12.0/19/25. The radius of the decay at the top (RD_T) is 8 cm and the radius of the decay at the bottom (RD_B) is 12 cm. Since decay is present at both ends, its length (LD) is considered to be 12 m. To shorten our notation, we will display the dimensions of the rot as 12.0/8/12 and call this the rot vector (as opposed to the log vector of 12.0/19/25).

Before we show you how to reduce the dimensions of this log to reflect the decay present, we will calculate the exact firmwood volume of the log to provide a baseline to which we can compare our efforts at reducing the log dimensions. In order to compute the exact firmwood volume of any log, you need to: (1) compute the total volume of the log; (2) compute the volume of the rot; and (3) subtract the rot volume from the total volume. This procedure works for any type of rot, but the approach to calculating rot volume changes with the rot type.

We can calculate the total volume and the rot volume of our log using either Smalian's formula directly or the numbers on the scaling stick. We use the latter approach here. The total volume of our log is:

12.0/19	680	
12.0/25	<u>1178</u>	
	1859	$\div 1000 = 1.859\ m^3$.

The rot volume of our log is:

12.0/8	121	
12.0/12	271	
	392	$\div 1000 = 0.392\ m^3$.

Thus, the exact firmwood volume of the log is $1.859 - 0.392 = 1.467\ m^3$.

We will now reduce the original dimensions to arrive at dimensions which approximate this volume. In this case, we have two choices: radii reduction and length reduction. Each of these procedures utilizes the "factors" that we discussed earlier in this lesson. We will illustrate each reduction method in turn using our example. You may find that these procedures look tricky, but they mimic the process followed when we determined the exact firmwood volume.

radii reduction

Radii reduction, as the name implies, consists of reducing the radii. The log length is kept exactly the same as the original measurement. The procedure consists of:

Step 1. Computing the net factor for each end of the log;
Step 2. Finding the radii which most closely correspond to each of these factors.

We will now go through these steps for our log. The net top factor for our log is:

Factor for R_T	= 19 cm which is:	113
− Factor for RD_T	= 8 cm which is:	20
		93

The net bottom factor is:

Factor for R_B	= 25 cm which is:	196
− Factor for RD_B	= 12 cm which is:	45
		151

The next step is to find the radii that most closely correspond to these net factors. The closest factor to 93 turns out to be the factor associated with a radius of 17 cm. (The factor for a radius of 17 cm is 91, while the factor for a radius of 18 cm is 102.) The closest factor to 151 is the factor associated with a radius of 22 cm. (The factor for a radius of 22 cm is 152, while the factor for a radius of 21 cm is 139.) Thus, the dimensions we would record for our log are 12.0/17/22.

As mentioned previously, these reduced dimensions will only approximate the exact firmwood volume. The volume associated with the reduced dimensions is:

12.0/17	545	
12.0/22	912	
	1457	$\div 1000 = 1.457\ m^3$.

Recall that the exact firmwood volume is $1.467\ m^3$. Answers you calculate will not always be this close, but the difference will not be large relative to the volume of the log.

Try calculating the exact firmwood volume and performing radii reduction on the sample logs below. Answers are provided for checking your results.

Log 1 Log vector (11.2/35/42); Rot vector (11.2/ 6/ 8)
Answers: Exact firmwood volume = 5.083 m^3
Reduced dimensions = 11.2/34/41
Reduced volume = 4.991 m^3

Log 2 Log vector (8.8/48/62); Rot vector (8.8/10/15)
Answers: Exact firmwood volume = 8.049 m^3
Reduced dimensions = 8.8/47/60
Reduced volume = 8.030 m^3

(***Hint:*** Some of the factors necessary for reducing dimensions for this log are not included on your scaling stick template. You will need to calculate these manually. This should not be too difficult if you go back in these notes to where factors were defined.)

length reduction

Length reduction, as the name implies, consists of reducing length. Radii are left as originally measured. The procedure consists of the following steps:

Step 1. Compute the average rot factor;
Step 2. Compute the average log factor;
Step 3. Compute the ratio of average rot factor to average log factor and multiply by log length;
Step 4. Subtract this quantity from the original log length to obtain the reduced log length.

The rationale behind length reduction is based on the proportionality of log length to log volume exhibited by Smalian's formula. Recall that we calculated exact firmwood volume as:

$$\text{Firmwood Volume} = \text{Total Volume} - \text{Defect Volume}$$

This formula can be rewritten as:

$$\text{Firmwood Volume} = \text{Total Volume} - \text{Total Volume} \times \frac{\text{Defect Volume}}{\text{Total Volume}}$$

Since log length L is proportional to log volume, we can substitute L for total volume. Similarly, since the ratio of average rot factor to average log factor approximates the ratio of defect volume to total volume, we can make that substitution as well. This produces:

$$\text{Reduced Length} = L - L \times \frac{\text{Average Rot Factor}}{\text{Average Log Factor}}$$

This is, of course, our formula for determining reduced length. In its simplest form, length reduction can be thought of as reducing the log length by a proportion equivalent to the proportion of defect in a log. Hence, length reduction works for any type of defect. An experienced scaler can usually approximate the proportion of defect present in a log quite quickly, often without needing to make detailed measurements.

In order to use length reduction on our example, we first need the average rot and log factors. The average rot factor for our log is:

	Factor for RD_T = 8 cm which is:	20
+	Factor for RD_B = 12 cm which is:	<u>45</u>
		65 ÷ 2 = 32.5.

The average log factor is:

Factor for R_T	= 19 cm which is:	113
+ Factor for R_B	= 25 cm which is:	196
		309 ÷ 2 = 154.5.

The reduced length is:

$$L - \frac{\text{Average Rot Factor}}{\text{Average Log Factor}} \times L = 12 - \frac{32.5}{154.5} \times 12 = 9.48$$

The dimensions a scaler would record are 9.4/19/25. (Note the rounding of the length down to 9.4 m because of the stipulation that length be recorded to the nearest 0.2 m.) These dimensions yield a volume of:

9.0/19	510
0.4/19	23
9.0/25	884
0.4/25	39
	1456 ÷ 1000 = 1.456 m^3.

This is almost exactly the same volume as we got using radii reduction (1.457 m^3). The two methods do not always agree as closely as this.

Log and rot dimensions identical to those on which you practiced radii reduction are given below. Try to apply the length reduction technique on these logs. Check your answers against those provided.

Log 1. Log vector (11.2/35/42); Rot vector (11.2/ 6/ 8)
Answers: Exact firmwood volume = 5.083 m^3
Reduced dimensions = 10.8/35/42
Reduced volume = 5.071 m^3

Log 2. Log vector (8.8/48/62); Rot vector (8.8/10/15)
Answers: Exact firmwood volume = 8.049 m^3
Reduced dimensions = 8.4/48/62
Reduced volume = 8.112 m^3

heart rot that does not extend through the log

If the **heart rot does not extend through the log** (i.e., it is visible at only one end), it is assumed to be cylindrical in shape. Assessment of defect length (LD) in this situation is strictly subjective; the scaler bases the assessment on experience gained from bucking and sawmill studies. For the purposes of this course, we will tell you the length of the rot.

Only length reduction is appropriate in this situation. Recall that radii reduction computes the reduced radii from net factors computed for each end of the log. It implicitly gives equal weight to each reduction. If no rot is present at one end, the reduction is zero. Hence, if LD < $^1/_2\,L$, radii reduction will result in too large a reduction, and if LD > $^1/_2\,L$, it will result in too small a reduction. Only if LD is exactly half of L will radius reduction work properly.

The only change that we need to make to the length reduction formula you learned earlier is to replace "average rot factor" with "rot factor" and L with LD. The formula becomes:

$$\text{Reduced Length} = L - \frac{\text{Rot Factor}}{\text{Average Log Factor}} \times \text{LD}$$

Again, we will illustrate this technique with an example.

The log vector for our example is 9.0/23/28. The rot vector is 3.0/0/9. This indicates that the defect length (LD in our notation) is 3.0 m, no rot is visible at the top end, and the rot has a radius of 9 cm at the bottom end of the log. Recall that exact firmwood volume is equal to the total log volume minus the rot volume. The total volume of this log is:

9.0/23	748
9.0/28	1108
	1856 $\div$ 1000 = 1.856 m^3.

The rot volume can be calculated using Smalian's formula or by using the appropriate factor. The factor associated with a rot radius of 9 cm is 25. Recall that the factor is the volume (in dm^3) of a 1 m cylinder with a specific radius. In order to determine the volume of rot in m^3 with an LD of 3.0 m, we multiply 25 by 3 and divide by 1000. This results in a rot volume of 0.075 m^3. Therefore, the exact firmwood volume for this log is 1.856 – 0.075 which equals 1.781 m^3.

In order to do length reduction, we need to determine the rot factor and the average log factor. The rot factor is 25 (RD_B = 9 cm). The average log factor is:

	Factor for R_T	= 23 cm which is: 166
+	Factor for R_B	= 28 cm which is: 246
		412 $\div$ 2 = 206.

Therefore, the reduced length is:

$$L - \frac{\text{Rot Factor}}{\text{Average Log Factor}} \times \text{LD} = 9.0 - \frac{25}{206} \times 3 = 8.64$$

Thus, the dimensions recorded would be 8.6/23/28. This yields a volume of:

8.0/23	665
8.0/28	985
0.6/23	50
0.6/28	74
	1774 $\div$ 1000 = 1.774 m^3.

You can practice this technique on the following logs and check your results against the answers provided.

Log 1. Log Vector (6.2/37/46); Rot Vector (4.0/ 0/ 8)
Answers: Exact Firmwood Volume = 3.314 m^3
Reduced Dimensions = 6.0/37/46
Reduced Volume = 3.285 m^3

Log 2. Log Vector (7.4/25/36); Rot Vector (6.0/ 0/14)
Answers: Exact Firmwood Volume = 1.864 m^3
Reduced Dimensions = 6.2/25/36
Reduced Volume = 1.871 m^3

butt rot

Butt rot is heart rot in the butt log of a tree. If it extends through the log, it is treated identically to heart rot. If the rot is visible only at the bottom end of the log, it is assumed to be conical in shape. The reason for assuming a conical shape is that decay present at the bottom end of a butt log is generally caused by root rot. Mill studies have indicated that the shape of root rot that extends up into the butt log tends to be conical. Length of decay can be approximated from the ratio of rot radius to total radius. Decay lengths for various ratios are given in Table 4.1.

TABLE 4.1 Assumed Rot Lengths for Butt Rot

RD/R	LD
0.0 to .25	0.9 m
.25	1.8 m
.25 to .50	2.7 m
.50	3.6 m
.50 to .75	4.5 m
.75	5.4 m
.75 to 1.0	6.3 m
1.0	7.2 m

Length reduction is again used for this type of rot. The only change in the formula from heart rot that does not extend through the log is that LD is divided by 3. This is to reflect the conical rot shape which is one third of the volume of a cylinder (the assumed rot shape for heart rot). The formula becomes:

$$\text{Reduced Length} = L - \frac{\text{Rot Factor}}{\text{Average Log Factor}} \times \frac{\text{LD}}{3}$$

As an example, consider a log with dimensions 10.0/13/18 and butt rot with a 9 cm radius. The ratio of RD_B to R_B is 9/18 which equals 0.5. You can see from Table 4.1 that the appropriate value for LD is 3.6 m. To calculate exact firmwood volume, we need the total log volume and the rot volume. The total log volume is:

10.0/13	265
10.0/18	<u>509</u>
	774 ÷ 1000 = 0.774 m³.

The rot volume is:

$$\frac{\pi\, \text{RD}_B^2}{10000} \times \frac{3.6}{3} = 0.031\,\text{m}^3$$

Therefore, the exact firmwood volume is 0.774 – 0.031 which equals 0.743 m³.

For length reduction, we need to determine the rot factor and the average log factor. The rot factor for a radius of 9 cm is 25. The average log factor is:

Factor for R_T	= 13 which is:	53
+ Factor for R_B	= 18 which is:	<u>102</u>
		155 ÷ 2 = 77.5.

Therefore, the reduced length is:

$$L - \frac{\text{Rot Factor}}{\text{Average Log Factor}} \times \frac{\text{LD}}{3} = 10 - \frac{25}{77.5} \times \frac{3.6}{3} = 9.61$$

Thus, the dimensions recorded would be 9.6/13/18. This yields a volume of:

9.0/13	239
9.0/18	458
0.6/13	14
0.6/18	30
	741 ÷ 1000 = 0.741 m³.

ring rot that extends through the log

As the name implies, **ring rot** looks like a ring in cross section (Figure 4.6). There is a solid core of wood inside the rot and a shell of solid wood outside the rot. Three radii need to be defined when ring rot is present: R_1 denoting the total log radius; R_2 denoting the radius out to the outer edge of the ring of rot; and R_3 denoting the radius out to the inner edge of the ring of rot. Ring rot may be present at one or both ends of the log. If it is present at both ends of the log, it is assumed to extend completely through the log.

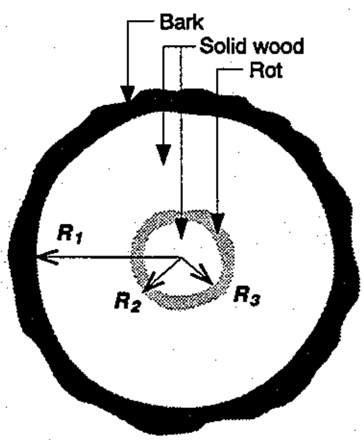


FIGURE 4.6 End view of ring rot in a log.

As in the case of heart rot that extends through the log, either radii reduction or length reduction is appropriate. We will demonstrate both kinds of reduction with an example.

It is convenient to display the relevant dimensions of a log with this type of rot using three vectors. The numbers of the vectors pertain to radii 1, 2 and 3 as defined above. Our example can be described as: vector 1 = (9.0/22/28); vector 2 = (9.0/15/20); and vector 3 = (9.0/12/15). The total volume of the log is calculated using the dimensions in vector 1 as:

9.0/22	684
9.0/28	1108
	1792 ÷ 1000 = 1.792 m³.

The volume using the dimensions in vector 2 is:

9.0/15	318
9.0/20	565
	883 ÷ 1000 = 0.883 m³.

This represents the volume of both the rot and the solid wood inner core. The volume using the dimensions in vector 3 is:

9.0/12	204
9.0/15	318
	522 ÷ 1000 = 0.522 m³.

This represents the volume of the solid wood inner core. Therefore, the volume of rot is 0.883 - 0.522 which equals 0.361 m^3. It follows that the exact firmwood volume is 1.792 - 0.361 which equals 1.431 m^3. The procedures used in radii reduction and length reduction mimic the steps we followed for calculating exact firmwood volume.

Radii reduction for ring rot requires that revised factors be determined for both ends of the log. The formula is:

$$\text{Revised Factor} = R_1 \text{ Factor} - R_2 \text{ Factor} + R_3 \text{ Factor}$$

The revised top factor for our example is:

Factor for R_{T1}	= 22 cm is	152
− Factor for R_{T2}	= 15 cm is	71
+ Factor for R_{T3}	= 12 cm is	45
		126

The reduced value for R_T is 20 cm. (The factor for 20 cm is 126 exactly). The revised bottom factor is:

Factor for R_{B1}	= 28 cm is	246
− Factor for R_{B2}	= 20 cm is	126
+ Factor for R_{B3}	= 15 cm is	71
		191

The reduced value for R_B is 25 cm. (The factor for 24 cm is 181; the factor for 25 cm is 196.) Hence, the dimensions a scaler would record for this log are 9.0/20/25. The volume associated with these dimensions is:

9.0/20	565
9.0/25	884
	1449 ÷ 1000 = 1.449 m^3.

Length reduction for ring rot requires that net top and bottom decay factors be determined prior to averaging the decay factors. The remainder of the procedure is identical to that described for heart rot that extends through the log. For our log, the net top decay factor is:

Factor for R_{T2}	= 15 cm is	71
− Factor for R_{T3}	= 12 cm is	45
		26

The net bottom decay factor is:

Factor for R_{B2}	= 20 cm is	126
− Factor for R_{B3}	= 15 cm is	71
		55

Therefore, the average net decay factor is (55 + 26) ÷ 2 which equals 40.5. The average log factor is:

Factor for R_{T1}	= 22 cm is	152
+ Factor for R_{B1}	= 28 cm is	246
		398 ÷ 2 = 199.

The reduced length is:

$$L - \frac{\text{Average Net Decay Factor}}{\text{Average Log Factor}} \times L = 9.0 - \frac{40.5}{199} \times 9 = 7.17$$

The dimensions that would be recorded by the scaler are 7.2/22/28. The associated volume is:

6.0/22	456
6.0/28	739
1.2/22	91
1.2/28	148
	1434 $\div 1000 = 1.434\ m^3$.

ring rot that does not extend through the log

In logs where ring rot does not extend through the log, only length reduction can be used. As with heart rot, the decay length needs to be estimated. The approach is similar to length reduction for ring rot that does extend through the log except that the ratio of average net rot factor to average log factor is multiplied by LD rather than *L*. The formula is:

$$\text{Reduced Length} = L - \frac{\text{Average Net Rot Factor}}{\text{Average Log Factor}} \times \text{LD}$$

WEIGHT SCALING IN BRITISH COLUMBIA

Scaling sticks are used to scale logs in flat booms or in dry land sorts. A small proportion of the quantity of timber scaled in the province is determined while the trees are still standing (**cruise-based scale**). We will not cover this scaling method in this course. The remainder of the wood scaled (quite a large proportion), is scaled using **weight scaling.**

The weight scaling that occurs in British Columbia is used to obtain an indirect estimate of scaled volume because stumpage rates are based on volume, not on weight. Truckloads of logs are weighed and a sample of these loads are scaled for volume, species and grade breakdown. The ratio of volume to weight on the sampled loads is used to convert the total weight of all the truckloads to volume.

The idea behind this approach is to capture the advantages of both stick and weight scaling. Volume can be most accurately scaled using a scaling stick. However, this is expensive and time consuming. Weight, on the other hand, is cheap and easy to measure. It is simply a matter of driving the truck onto a scale. There should be a good relationship between weight and volume. Selecting a suitable proportion of the truckloads to stick scale should provide a ratio for converting weight to volume with acceptable accuracy.

The sampling technique used to calculate the scaled volume of wood and its sampling error is called **ratio estimation** (also known as ratio of means). We will not provide the formulae for this sampling method because their use is beyond the scope of this course. Ratio estimation and a number of other sampling techniques are covered in the course Forestry 238 'Forest Mensuration'.

LOG GRADING

Log grading is the process of assigning logs to various classes on the basis of surface characteristics and the amount of defect present. The grade of a log should reflect the quality of the log in terms of possible end use, and hence the log's value. Factors commonly considered in assigning log grade are log size (length and top radius), presence of defects, prevalence of knots, and growth rate (determined by the number of growth rings in some unit length). Grading guidelines vary among jurisdictions. As an example of one set of grading standards, we will look briefly at grading within British Columbia.

Log grading is of concern only within the coastal region of British Columbia. In the interior of the Province, logs are divided into only "small" and "large" log categories. There are several reasons for the differences in grading practice. The principal reason is probably historical precedent. The forest industry of the coast developed around the old-growth forests with very large and potentially valuable logs. The predominant product was lumber. Concentration of logging effort in a relatively small region led to much buying and selling of logs among companies. Among logs from old-growth trees, there are considerable quality differences that affect their end use. Thus it became important to recognize these differences and agree on standards. In contrast, forest companies developed much more in isolation from one another in the interior of the province. There never was much buying or selling of logs; most companies simply cut timber for their own use. Also, most logs were much smaller and of more uniform quality than those of the coast.

The coastal grades in British Columbia vary by end use. Product categories recognized are: (1) peelers (for veneer); (2) lumber; (3) sawlogs; (4) shingle; and (5) pulp. Not all products are appropriate for each species. For example, the shingle category is applicable to only western redcedar logs. Except for lumber which has four grades, all other categories have three grades. Not all of these grades are appropriate for a given species. Table 4.2 contains the grade distribution by species and products. Table 4.3 contains the specifics of No. 3 Peeler Douglas-fir, Grade Code C. This information is provided as examples of grading requirements. You are not expected to remember the details of either of these tables.

TABLE 4.2 Log Grades by Species and Use

(*Source:* S. Watts (editor). 1983. *Forestry Handbook for British Columbia*, 4th ed. Forestry Undergraduate Society, Faculty of Forestry, University of British Columbia.)

Species	Peelers			Lumber				Sawlogs			Shingle			Pulp		
	A	B	C	D	E	F	G	H	I	J	K	L	M	X	Y	Z
Douglas-fir	•	•	•	•				•	•	•				•	•	•
Cedar				•		•		•	•	•	•	•	•	•	•	•
Hemlock				•				•	•	•				•	•	•
Balsam			•	•				•	•	•				•	•	•
Cypress				•	•	•	•	•	•	•				•	•	•
Pine				•				•	•	•				•	•	•
Spruce			•	•	•	•	•	•	•	•				•	•	•

TABLE 4.3 Criteria for No. 3 Peeler Douglas-fir, Grade Code C.

(*Source:* S. Watts (editor). 1983. *Forestry Handbook for British Columbia*, 4th ed. Forestry Undergraduate Society, Faculty of Forestry, University of British Columbia.)

Grade Rule: Logs 5.2 m or more in length and 19 cm or more in radius where at least 80% of the gross scale will cut out on a rotary lathe into veneer.

Log Requirements to Make the Grade:

(a) No conk, conk stain or pocket rot is permitted.
(b) There must be no fewer than 5 annual rings in each 2 cm of diameter.
(c) No knots over 4 cm in diameter are permitted and knots 4 cm or less in diameter must be well spaced.
(d) Maximum twist permitted over 30 cm of length is 14% of the radius up to a maximum deviation of 8 cm.
(e) Other defects are permitted as listed for No. 1 Peeler Fir, see items (e) to (m).

REVIEW/SELF-STUDY QUESTIONS

Do these questions before you go on to complete the Graded Assignment. These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. What is scaling?
2. Why do different jurisdictions have different scaling procedures?
3. Why are logs scaled?
4. Identify three basic units of measurement for scaling.
5. What is a board foot log rule?
6. What are the reasons for the relatively recent shift in Canada to the use of cubic volume units?
7. Why was Smalian's formula chosen for calculating log volumes in British Columbia?
8. What are the procedures and criteria for measuring log radius and length in British Columbia?
9. Explain why the half cylinder volumes displayed on the scaling stick can be used to provide the same volume for a log as Smalian's formula.
10. What is a "factor" and for what can it be used?
11. Why can length reduction be used for any type of defect while radii reduction is valid only if the rot extends through the log?
12. Describe the weight scaling procedure used in British Columbia. Why is it used?
13. What is log grading?
14. What factors are commonly considered in assigning logs to a grade?

Lesson 5

Introduction to Photogrammetry and Photo Interpretation

INTRODUCTION

Lesson Overview

This lesson provides an introduction to photogrammetry and photo interpretation. These terms will be defined and examples of their uses will be provided. The concept of vertical aerial photography, characteristics of light, and the photographic process will be summarized. A good understanding of the material covered in this lesson is essential for Lessons 6, 7, and 8.

Lesson Objectives

After completing this lesson and the assignment, you should be able:

1. to distinguish between photogrammetry and photo interpretation;
2. to explain the basic principles of the photographic process;
3. to sketch the geometry of vertical aerial photography and to use it to explain scale;
4. to describe the principles of stereoscopic vision;
5. to view your aerial photographs stereoscopically;
6. to prepare aerial photographs for long-term stereoscopic viewing.

Lesson Readings

Some of the material presented in this lesson may be found in Avery and Berlin, pages 1–48. Some of this material is covered in Avery and Burkhart, pages 253–258.

Lesson Assignment

When you have completed this lesson, answer the self-study questions at the end. You should then complete Graded Assignement #4 and mail it to your tutor by the date indicated on your course schedule. Be sure to include a pink assignment cover sheet.

BACKGROUND TO PHOTOGRAMMETRY AND PHOTO INTERPRETATION

Photogrammetry is derived from three Greek words: *photos* meaning light; *gramma* meaning that which is drawn or written; and *metron* meaning to measure. If you put these meanings together, you get a good definition of photogrammetry: "Measurement of that which is drawn or written with light." The American Society of Photogrammetry provides a more formal definition:

> "Photogrammetry is the art, science, and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring, and interpreting photographic images and patterns of electromagnetic radiant energy and other phenomena."

Photo interpretation involves identifying objects on photographs and determining their meaning or significance. It is sometimes considered a subset of photogrammetry. In this course, we use **photogrammetry** to refer to the measurement aspects and **photo interpretation** to refer to the interpretive aspects of obtaining information from aerial photographs.

Most people have some experience with photogrammetry and photo interpretation from standard photography. Suppose that someone shows you a family snapshot. You recognize the individuals in the picture and perhaps the locale. For example, if the people in the picture were wearing swimming suits and standing on sand, you might conclude that they were at the beach. All this is photo interpretation. You also can make some observations with respect to some memory standard you have of the individuals. For example, you may conclude that so-and-so is looking older or putting on weight. These are measurements of a sort, but they are subjective since you have no definite, reproducible standard. Without such standards, you can be fooled easily. For example, size is difficult to determine on a photograph without a recognizable context. You can also be fooled about colours because of lighting, film processing, and so on.

Aerial photographs can be regarded in the same way as snapshots. They can be used to identify landforms and artificial features, to distinguish between forest types, and so on. These uses can all be classified as qualitative. However, it is also possible to make quantitative assessments. This is done by noting (or controlling) conditions under which the aerial photographs are taken. If these conditions are suitable, it is possible to make quite accurate measurements from a single photo or a stereo pair of photos.

Applications of Aerial Photography

Aerial photography is widely used in forestry and many other fields. Some of the reasons for this include:

- It is a labour saving tool. Traditionally, it has been a cost-effective way to do forest planning. In the past few decades, it has become a cost-effective way to obtain certain mensurational measurements.
- It is essential for map making and navigation. In the early part of this century, all map making was done by ground surveys. With the advent of aerial photography, the trend began to shift towards using this information for constructing maps. Today, almost all map making is done using aerial photography and/or satellite imagery.

- It provides a ready means for detecting changes. Change detection ranges from before/after photos taken from the same position to comparing current aerial photographs to previously flown photographs, perhaps with different scales and photo locations.

- It enables the window of perception to be expanded beyond that of human vision. Standard photography extends slightly into the infrared region of the spectrum. Certain objects can be more easily distinguished in these bands than by using only visible bands. If nonphotographic techniques are employed (e.g., radar, thermal scanners), energy bands well outside the range of human vision can be sensed.

- It has recreational applications. Aerial photographs provide a visible image of ground terrain. Although these photographs are not maps (we will discuss this in later lessons), they provide a means of navigation useful when hiking, hunting, and so on. It is possible for the public to order photographs of most areas in British Columbia and elsewhere in Canada.

- It can be quite profitable. Aerial photographs are used in geology (e.g., mineral/oil exploration), in court cases (e.g., oil spills, SO_2 damage), for military strategy (e.g., spy satellites, reconnaissance), and many other applications.

Some of the general uses of aerial photography in forestry include:

- *Measuring heights of objects.* With the photographs included with your course materials, it is possible to discern height differences on the order of the height of the curbs along a city street. This requires stereoscopic viewing and expensive equipment. In the assignment associated with Lesson 6, you will be measuring heights using a pocket stereoscope and a stereometer that work according to the same principles as more expensive instruments, although they cannot attain the same degree of precision.

- *Planimetric mapping.* This consists of determining the horizontal ground position of objects. With the photographs in your course materials, you can obtain accuracy to the same standards as surveyed maps for considerably less cost. This is achieved through stereoscopic viewing with simple instruments. More expensive equipment is not necessarily more accurate, but is faster to use. You will have an opportunity to practice mapping in the assignment associated with Lesson 7.

- *Topographic mapping.* This consists of determining both the horizontal and vertical dimensions of the position of objects. Topographic mapping requires stereoscopic viewing and expensive equipment. We will touch on this topic briefly in Lesson 7.

- *Photo Interpretation.* Forestry applications include stand typing, species identification, landform analysis, watershed analysis, and vegetation damage assessment. We will cover some of these aspects in Lesson 8.

Vertical Aerial Photography

In order to explain what vertical aerial photography is, we first need to introduce some terminology. Some of the terms to be introduced are illustrated in Figure 5.1.

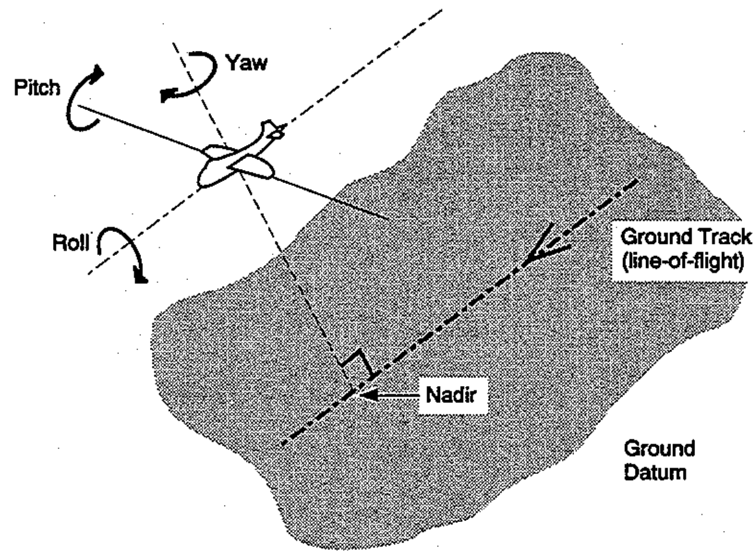


Figure 5.1 Illustration showing terminology used in photogrammetry.

Picture an aircraft flying above the ground surface. Since the ground is seldom flat, and a flat surface is useful to the measurements we will subsequently discuss, we will create an imaginary flat surface at some useful elevation for the vicinity in which our aerial photographs are taken. This surface is called a **ground datum.** We will express the flying height of the aircraft with respect to the ground datum for the remainder of this course unless otherwise specified. The point directly below the aircraft is called the **nadir.** As the airplane flies over the ground datum, the nadir moves along the surface forming an imaginary line depicting the path of the plane. This line is called the **line-of-flight** and is generally abbreviated as **l-o-f.**

In order to determine the orientation of the aircraft, three axes are required. One extends perpendicularly from the ground datum and passes vertically through the aircraft. Any rotation of the airplane around this axis is called **yaw.** Another axis extends through the aircraft from nose to tail. Rotation around this axis is called **roll.** The third axis extends through the wings. Rotation around this axis is called **pitch.** Together, the magnitudes of roll, pitch, and yaw determine the **attitude** of the aircraft in flight.

Assume the aircraft is carrying a downward-pointing camera. Periodically, the camera takes a photograph of the ground surface. The centre of this photograph is called the **principal point** (PP). More precisely, the principal point is the point where a perpendicular projected through the centre of the lens intersects the photo image. In other words, it is an extension of the optical axis of the camera.

If you look at any one of the aerial photos provided in your kit you will see marks at the midpoint along each of the sides. These marks are built into the camera and are called **fiducial marks.** The fiducial marks are used to determine the exact centre of an aerial photograph (i.e., the principal point). The centre is found by connecting the centres of the opposing fiducial marks with straight lines. These lines intersect at the principal point.

Standard aerial photographs are taken in such a manner that there is about 60 percent overlap between consecutive photographs along a line-of-flight. There is approximately 10 percent overlap between lines-of-flight. The overlap which exists within a line-of-flight means that the principal point on one photograph will also appear on the photographs on either side of it, as illustrated in Figure 5.2. The image of a principal point on an adjacent photograph is known as a **conjugate principal point** (CPP). Once conjugate principal points and the principal point are marked on an aerial photograph, the l-o-f can be approximated since you can be sure of at least three locations over which the aircraft flew. How to mark conjugate principal points and the l-o-f on your photographs are described in the assignment for this lesson.

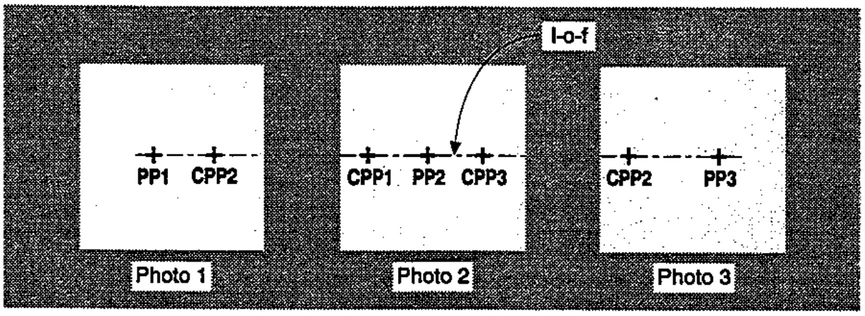
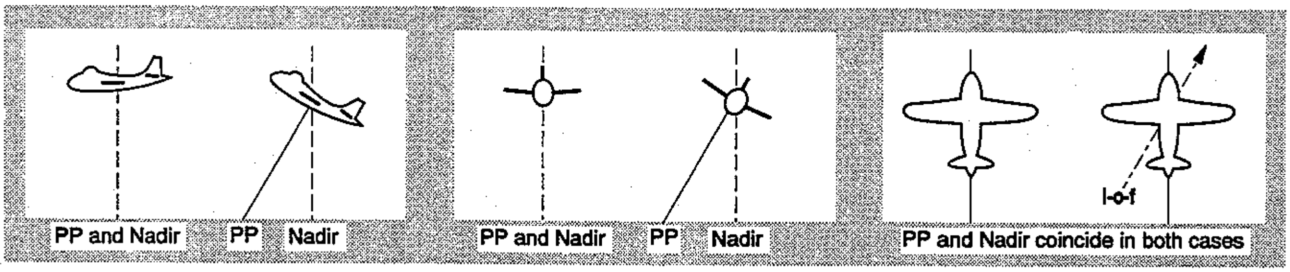


FIGURE 5.2 Relationship between principal points and conjugate principal points.

There is also a nadir on the photograph. Recall that this is the point directly beneath the camera centre at the time of the exposure. The nadir and the principal point may or may not coincide, depending on the attitude of the aircraft. It is easiest to see the effect if you look at pitch, roll, and yaw individually. You can see from Figures 5.3 and 5.4 that the presence of both pitch and roll cause the principal point to deviate from the nadir. However, no matter how much yaw is present, the nadir and principal point coincide as long as there is neither pitch nor roll (Figure 5.5).



Case 1: Pitch = 0 Case 2: Pitch ≠ 0

Case 1: Roll = 0 Case 2: Roll ≠ 0

Case 1: Yaw = 0 Case 2: Yaw ≠ 0

FIGURE 5.3 Effect of pitch (assuming roll = yaw = 0).

FIGURE 5.4 Effect of roll (assuming pitch = yaw = 0).

FIGURE 5.5 Effect of yaw (assuming pitch = roll = 0).

Truly vertical aerial photographs are defined as photographs in which the nadir and principal point coincide. It is necessary to know the location of the nadir for making measurements on an aerial photograph. However, there is no easy way to find the nadir unless it happens to coincide with the principal point which is easy to find. Hence, it is useful if the photographs are truly vertical. Using truly vertical photographs also simplifies some of the geometry that underlies the formulas we will later develop for you. We will consider only truly vertical aerial photographs in this course.

Tilt is said to be present if pitch and/or roll are not zero. The presence of tilt can be seen qualitatively from an image on one of the edges of the photograph. There is a series of circles in which you may be able to see a bubble. This is the equivalent of a spirit level; the closer the bubble is to the centre in the circles, the more level the flight. Modern aircraft employ an independent camera mount enabling truly vertical aerial photography to be achieved on an operational basis. However, things can still go wrong so it is useful to specify a maximum amount of acceptable tilt if you are contracting your own aerial photographs. The present day standard is usually less than 1 degree.

Yaw, caused primarily by crosswind, is almost always present during a flight. The presence of yaw means that the aircraft is not flying in the same direction it is pointing. The impact on aerial photographs is that the l-o-f does not usually pass through the fiducial points, making it more difficult to locate the l-o-f.

FACTS OF LIGHT

Photography as a Measurement Process

In photogrammetry, photography is viewed as a measurement process. A logical question to ask is, "What is it that we measure?" This can best be answered by addressing two subquestions:

Q_1: "What determines **where** things appear on an aerial photograph?"

Q_2: "What determines **how** things appear on an aerial photograph?"

The answer to Q_1 lies in the field of geometry. The answer to Q_2 lies in the field of radiometry. Photogrammetry emphasizes Q_1 and photo interpretation emphasizes Q_2. A general understanding of light energy and the photographic process is necessary to give you a proper perspective when you are studying either of these areas.

The EMR Spectrum and Visible Light

Visible light is a form of **electromagnetic radiation** (EMR). In fact, visible light forms only a very small portion of a vast spectrum of energy (Figure 5.6). The EMR spectrum extends over 16 orders of magnitude. It ranges from gamma rays that can be smaller than 0.3 pm (1 pm = 1×10^{-12} m) to television and radio waves that can be larger than 10 km (i.e., > 1×10^{4} m). The visible range encompasses only the region between 400 and 700 nm (1 nm = 1×10^{-9} m). You may also see this range expressed as being from 0.4 to 0.7 μm. (A μm is called a micrometre or a micron. One micron equals 10^{-6} m or one thousandth of a mm.)

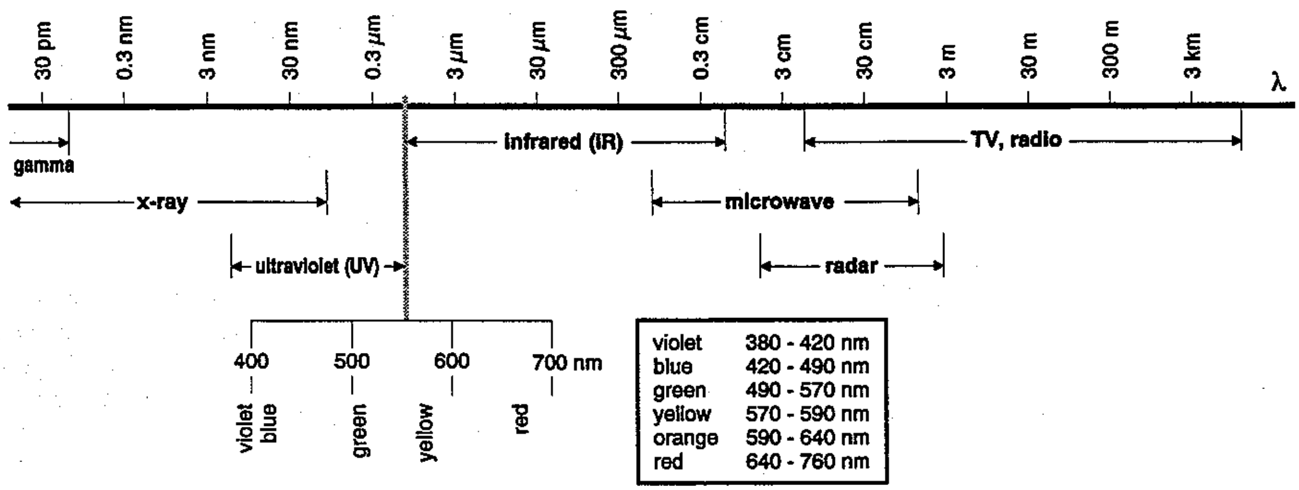


FIGURE 5.6 The electromagnetic radiation (EMR) spectrum.

The atmosphere absorbs or reflects most EMR wavelengths. This process is called **atmospheric attenuation.** It is a good thing for us that this occurs since many of the energy types, particularly those with shorter wavelengths, are damaging or deadly to us. Recent concerns over the depletion of the ozone layer is related to a reduction in atmospheric attenuation. The quantity of particular wavelengths absorbed or reflected at any point in time is related to atmospheric conditions.

Certain wavelengths are essentially passed intact. These are known as **pass bands.** Some of the important pass bands include:

- *Visible light and just into the ultraviolet band (0.3 to 0.7 μm).* Our eyes have adapted to make use of energy on these wavelengths. The small portion of ultraviolet that penetrates is what causes suntans.
- *Reflected and emitted thermal infrared (10 to 14 μm).* This is important because it allows the sun's energy that has been absorbed by the earth to be released through the atmosphere as heat. When free release of the heat does not occur, for example in a greenhouse, the temperature increases. Particular atmospheric pollutants can cause the same effect on a global scale. This process has been called the **greenhouse effect.**

Normal aerial film is sensitive to wavelengths of 0.3 to 1.1 μm, a range that is slightly greater than the range of wavelengths that human vision can perceive. In aerial photography, the lower end of the range is frequently cut off by filters. You will notice that film sensitivity extends slightly into the infrared (from 0.7 to 1.1 μm). This is only a small portion of the large infrared band, which extends from 0.7 to 3000 μm. Thermal infrared (emitted energy) has wavelengths between 2.5 and 14 μm, so there is no such thing as thermal infrared film. However, there are certain non-photographic sensors that can sense wavelengths in this region. We will cover a few of them in Lesson 8. The region of the infrared captured by film is termed the **near infrared** and is comprised of reflected infrared radiation.

Certain sensing devices supply their own sources of EMR (e.g., radar and X-rays). These types of sensors are called **active sensors** for this reason. **Passive sensors** rely on outside sources of radiation (e.g., standard aerial photography). A standard terrestrial camera when used without a flash is a passive sensor. When it is used with a flash it becomes an active sensor. Passive sensors are limited by the characteristics of the surface illumination.

Spectral Response Curves

When EMR falls on a surface, three things happen which are relevant to the portion of the EMR spectrum that is recorded on film: reflection, transmission, and absorption. Figure 5.7 shows EMR striking a leaf's surface. A fourth component, emission, is the ultimate response to absorption; however, the wavelengths involved are outside the sensitivity range of film.

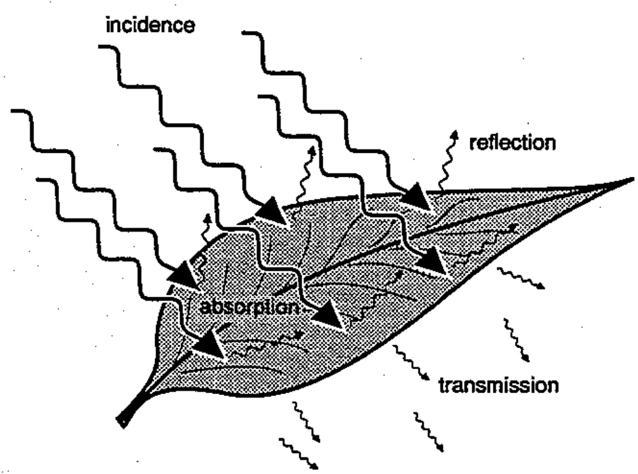


Figure 5.7 EMR striking a leaf.

The relative amount of each of these components depends upon the wavelength (λ). This is connected with the idea of a **spectral response curve.** A spectral response curve relates percentage of energy reflected to the EMR wavelength. (See Figure 5.8 for an example.) Every object has a characteristic spectral response curve known as a **spectral signature.** This can be used to identify unknown materials in a manner equivalent to finger printing. In fact, there is a

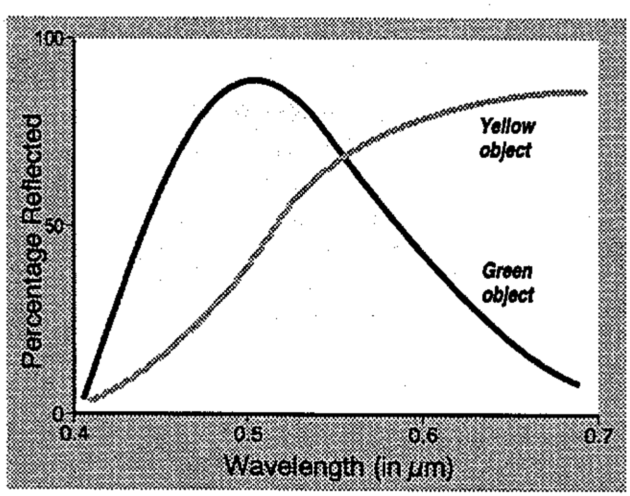


Figure 5.8 Spectral response curves of two objects, one yellow and the other green.

branch of analytic chemistry known as spectroscopy which uses spectral signatures under laboratory conditions to identify chemical components.

This leads us to the question, "Can we measure the spectral response curve of images using aerial photographic techniques to identify objects on the ground?" Unfortunately, this idea doesn't directly extend to aerial photography. Let's look at some of the reasons in a little detail.

The first major reason why spectral signatures cannot be used to exactly identify objects on the ground using aerial photography is because the process is passive. This means that we do not control the characteristics of the scene illumination. Since the spectral response curve relates percentage of energy reflected to wavelength, not to the absolute amount of energy present, the **spectral distribution** of the source illumination must be taken into account. For example, sunlight at the equator has a different spectral distribution than sunlight at northern latitudes. Atmospheric conditions also influence the spectral distribution of sunlight. The spectral distributions of several different light sources are given in Figure 5.9.

A possible solution to this difficulty is to make the film respond inversely to the illumination characteristics of the source illumination. This would result in recorded results that are true spectral signatures of the surface material. There have been some broad attempts to do this with retail (terrestrial) films. It is possible to buy "daylight," "tungsten light," and "fluorescent light" films among others. However, these are developed for average conditions and likely won't exactly match the specifics of any particular lighting condition. Aerial photography uses filters to control some aspects of the scene illumination. However, as with using different film types, the filters are not likely to exactly match the specifics of any particular lighting condition.

A second reason why spectral response curves cannot be used to identify objects on aerial photographs is that certain sensors respond differently to different wavelengths. For example, we all respond differently to different colours. A standardized curve was developed for photographic purposes called the **standard observer curve.** This curve (Figure 5.10) shows the average human sensitivity to various wavelengths of visible light. Sensitivity can be

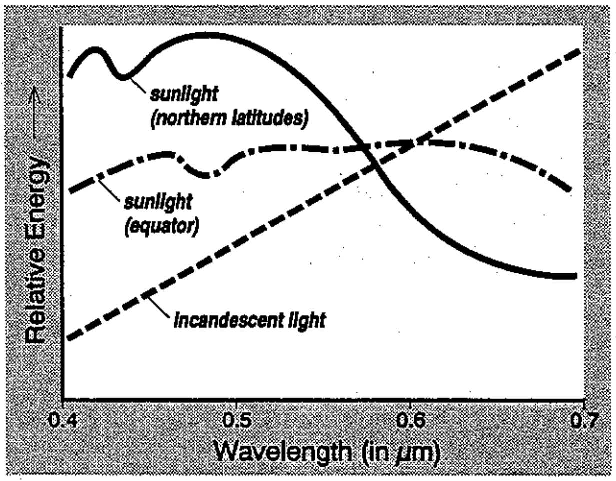


FIGURE 5.9 Spectral distribution of several different light sources.

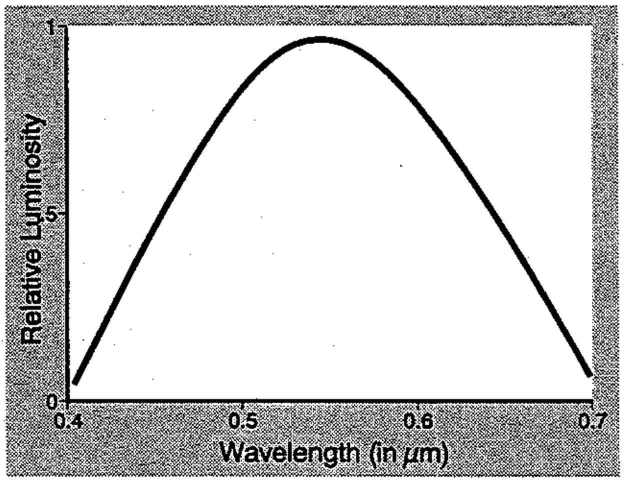


FIGURE 5.10 The Standard Observer (Photopic) Curve.

thought of as the ability of the light to create a sense of brightness. The standard observer curve is used as a weighting in black and white film so that the film reflects the sense of brightness that the average human sees.

We can now return to one of the questions posed earlier in this lesson, "What determines *how* something appears in an aerial photograph?" The answer is:
- the spectral response characteristics of the surface material (i.e., the spectral response curve);
- the spectral characteristics of the surface illumination;
- the spectral sensitivity of the sensor including humans directly and humans and film together;
- certain other factors including topography and atmospheric conditions.

CAMERAS, FILMS AND FILTERS

THE PHOTOGRAPHIC SYSTEM

The photographic system is comprised of the camera, lens, film, and filter. These components act in concert to produce the conditions that make photography possible. The film and filter serve the following purposes:
- to select the response characteristics that help delineate features of interest. Usually filters are added to alter response since it is easier than having a special film designed.
- to extend the window of response beyond that of human vision. Recall that film can sense EMR wavelengths into the near infrared (up to 1.1 μm) while humans can see EMR wavelengths only between 0.4 and 0.7 μm. Certain objects (e.g., coniferous vs. deciduous forest cover) can be more easily distinguished in the near infrared band.
- to sense and store the image. Many other sensing systems use film as a means of storing or displaying sensed data. However, the photographic process is the only sensing system that uses film as both the sensing and storage medium.

The camera and lens components serve the following purposes:
- to control exposure parameters (aperture and time). The amount of light that gets through to the film is a function of certain characteristics of the camera and the lens interacting together, as you will soon see. The amount of light needs to be controlled if the image is to turn out properly.
- to produce an image with regular geometry. If the images on aerial photographs are not distorted, then relating measurements of features on the photographs to features on the ground is much easier. Standard aerial photographs can be assumed to have no distortion due to the lens.

RELATIVE APERTURES

Several factors determine how much light reaches the film. These are:
- characteristics of the source illumination. The brighter the conditions, the more light that reaches the film.
- the **aperture** of the lens. This is the effective diameter of the lens when the shutter opens. The larger the aperture, the more light that reaches the film. The amount of light entering the lens opening is linearly related to lens area.

Since the lens area is proportional to the square of the lens diameter, therefore the amount of light reaching the film is proportional to the square of the lens diameter.

- the **focal length** of the lens. This is the distance between the camera lens centre and the film. The longer the focal length, the less light that reaches the film. This can be explained in a number of ways. Perhaps the easiest way to understand this relationship is to remember that images are larger with a longer focal length. This means that light reflecting from the objects has to cover a larger area of film. Since the size of the image is inversely proportional to the focal length, it follows that the light reaching the film is also inversely proportional to focal length.
- exposure time. This is the length of time the shutter stays open. The longer the exposure time, the more light that reaches the film.

The relationship between some of these factors is illustrated in Figure 5.11.

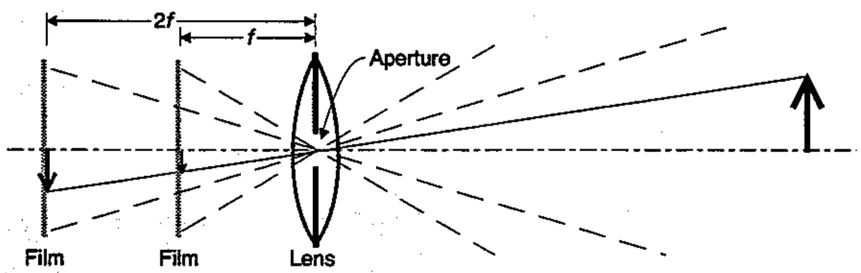


FIGURE 5.11 Relationship among focal length (*f*), size of image, and view angle.

There is also a relationship between the view angle of the lens and the focal length. For a fixed size of film, shorter focal lengths imply a wider angle lens. Longer focal lengths imply a narrow angle lens. Focal lengths for typical lenses used in aerial photography are: (1) narrow angle (305 mm); (2) normal angle (210 mm); and (3) wide angle (152 mm).

Relative aperture (also known as ***f*/stop**) determines how much light reaches the film per unit of time under some known lighting condition. It combines the effect of aperture size and focal length to facilitate rapid setting of exposures. Another way of looking at *f*/stops is to think of them as measures of the "speed" of the lens that are independent of focal length. A definition of *f*/stop is the focal length of the lens divided by the lens diameter.

In order to see how the process works, let's address the question, "Does an *f*/stop of *f*/1 represent more or less light than an *f*/stop of *f*/2?" A good way to answer a question like this is to think up some appropriate numbers and reason from them. For example, a lens with a focal length of 40 mm and a lens diameter of 20 mm represents an *f*/stop of *f*/2 (40 ÷ 20). In order to get an *f*/stop of *f*/1, the effective diameter of the lens must be increased to 40 mm or the focal length must be halved to 20 mm. Either way, the amount of light reaching the film per unit time will increase. Hence, an *f*/stop of *f*/1 represents more light than an *f*/stop of *f*/2.

Camera *f*/stops are set up in such a way that each "step" down halves the amount of light. Since the amount of light captured is proportional to the area of the lens, and area is proportional to the square of the diameter, decreasing the diameter of the lens by a factor of $\sqrt{2}$ (which approximately equals 1.4) will halve the amount of light captured. A listing of *f*/stops and an associated speed index is given in Table 5.1.

TABLE 5.1 Relationship between Relative Aperture and Speed Index

Relative Apertures:												
f/1	1.4	2	2.8	4	5.6	8	11	16	22	32	45	64
Speed Index:												
1	2	4	8	16	32	64	128	256	512	1024	2048	4096

The speed index and *f*/stops are used together to allow the interchanging of lenses and/or the adjustment of relative apertures while still maintaining constant exposure. Since the amount of light is halved with each increase in *f*/stop number, the time the shutter is open must be doubled to achieve the same exposure. If you step down four *f*/stops (e.g., from *f*/2 to *f*/8), the exposure time must be extended 16 (i.e., 2^4) times to maintain the same exposure. You can see this from the table by comparing the speed index for *f*/2 (4) and *f*/8 (64).

Check to see if you understand this concept by trying this question from an old examination.

> Harvey 'Flash' Kirk, ace photographer, determined that the correct exposure for his picture at f/1.4 was 1/125 seconds using his standard 55 mm lens. At the last minute, Harvey decides to use his 135 mm telephoto lens instead. This lens has a maximum relative aperture of f/2.8. What is his correct exposure time?

Hint: The sequence of exposure times on a standard camera (in seconds) are: 1/1000, 1/500, 1/250, 1/125, 1/60, 1/30, 1/15, 1/8, 1/4, 1/2, 1.

Answer: 1/30 second.

SCALE ON VERTICAL AERIAL PHOTOGRAPHS

Scale may be defined as the ratio between a model representation of an object or distance and the real object or distance. You are probably most familiar with scales from maps. Almost all maps have scales that tell you how to convert distances on the map to actual distances on the ground. If you have looked at several maps, you probably have realized that there is more than one way of presenting a scale.

We can think of photographic scale as equaling a/A where a is a distance on a photograph and A is the corresponding ground distance. It can be proven geometrically that triangle Wxy in Figure 5.12 is similar to triangle WXY. From

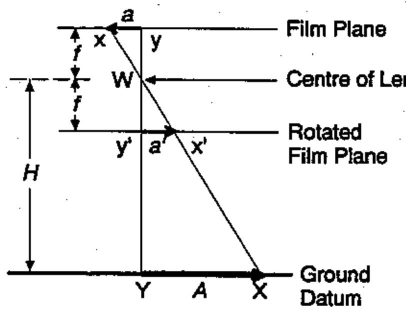


FIGURE 5.12 Vertical aerial photography diagram.

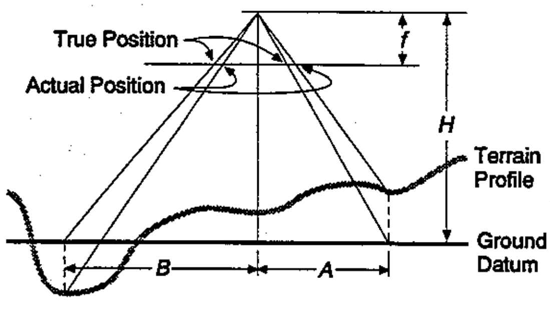


FIGURE 5.13 The effect of terrain changes on scale.

the properties of similar triangles, it follows that $f/H = a/A$ where H is the flying height above ground datum. Hence, we can define scale in this diagram as being equal to f/H.

We have also used Figure 5.12 to illustrate a relationship that we will employ in future diagrams of this type. Obviously, the true film plane lies on the opposite side of the lens from the ground. This results in a reversal of images. It is more convenient, and exactly equivalent geometrically, if triangle Wxy is rotated through 180 degrees to become triangle Wx'y'. This will offset the image reversal while maintaining geometric integrity.

Scale varies across aerial photographs if the terrain is not absolutely flat. We will discuss this feature, termed **displacement due to relief**, in detail in the next lesson. It will suffice at the present time for you to think of the scale at any point on the photograph as being related to the vertical distance between the aircraft and that point. As this distance changes with the topography, the scale changes. In other words, there will likely be many scales on a single aerial photograph. We have tried to illustrate this principle in Figure 5.13.

In order to have a single representative scale for an aerial photograph, the concept of a **nominal scale** is used. Nominal means existing in name only. The nominal scale of an aerial photograph is simply f/H. If the ground datum is established close to the same elevation as the terrain and the terrain is relatively flat, then the nominal scale will be close to the actual scale at any point on the photograph.

The present trend in aerial photographs is to use a unitless scale called a **representative fraction** or RF scale. Common nominal scales for post-metric conversion Canadian aerial photographs are 1:10,000 and 1:20,000. The photographs in your lab kit have a nominal scale of 1:10,000. A scale of 1:10,000 means that 1 of any unit distance on the photograph is equal to 10,000 of these units on the ground. For example, 1 mm represents 10,000 mm or 10 m on the ground.

In many older aerial photographs, flight paths were controlled so that the nominal scales made sense in imperial units. For example, many photos used to be flown so that the nominal scale was 1 inch = ¼ mile. This translates to 1:15,840 using an RF scale.

You may have difficulty remembering which of two scales is larger when the scales are given as RF's. The key to remembering is to realize that the second number in an RF scale is really a denominator in a fraction. The larger the number in the denominator, the smaller the fraction, and the smaller the scale. For example, 1:10,000 is a larger scale than 1:20,000.

For the photographs in your lab kit, ground datum was established at mean sea level (MSL), flying height was 3050 m above MSL, and focal length was 305 mm. This translates to a nominal scale of:

$$S = \frac{f}{H} = \frac{305\ \text{mm}}{3050\ \text{m}} = \frac{305\ \text{mm}}{3050000\ \text{mm}} = \frac{1}{10000}$$

Suppose the height above MSL for a point on the photographs is 120 m above MSL. The scale at this point would be:

$$S = \frac{f}{H} = \frac{305\ \text{mm}}{(3050 - 120)\ \text{m}} = \frac{305\ \text{mm}}{2930000\ \text{mm}} = \frac{1}{9600}$$

A further complicating factor for determining actual scale at some point on the photograph is not knowing the exact flying height. The specified flying height above ground datum is easily calculated from knowing the nominal scale and focal length. However, the actual flying height above ground datum for any photograph may differ from this because it is difficult for the pilot to fly at a constant elevation. Differences in flying height of up to 10% of the specified flying height are sometimes encountered.

STEREOSCOPIC VISION

Stereoscopic vision refers to our ability to see in three dimensions. In this section we describe how humans perceive depth, then explain how a stereoscope helps you to view two different images of an object stereoscopically. We conclude by describing how an overlapping pair of aerial photographs can be viewed stereoscopically.

Perceiving Depth

We see things in three dimensions because we have two eyes. Our concept of depth is a result of the difference in convergence angles between our eyes when they focus on the front (top) and rear (base) of an object (Figure 5.14). Everyone does not have the same sense of depth because the distance between people's eyes (known as **inter-pupilary distance** or IPD) differs. The "average" IPD is 65 mm.

Normal people can resolve a difference between θ_2 and θ_1 on the order of about 20 seconds of a degree. Given that there are 60 minutes in a degree and 60 seconds in a minute, 20 seconds of a degree translates to 1⁄180th of a degree or 0.00556 degrees. Some individuals can recognize a difference of about 10 seconds of a degree.

Cameras can be used to take the place of your eyes. If two cameras are set up at right angles to an object along a common baseline separated by a distance that will allow some portion of their fields to overlap, photographs of the object from each of the cameras are analogous to what your eyes might see (Figure 5.15). Movies that are in 3-D are shot in this manner. When the photographs are properly aligned, each of the photographs presents a view of the object from the same perspective as you would see it if each of your eyes were in the position from which the respective photographs were taken. If you are able to focus each eye on the object in the corresponding photograph simultaneously, the two images should merge into a three dimensional image of the object.

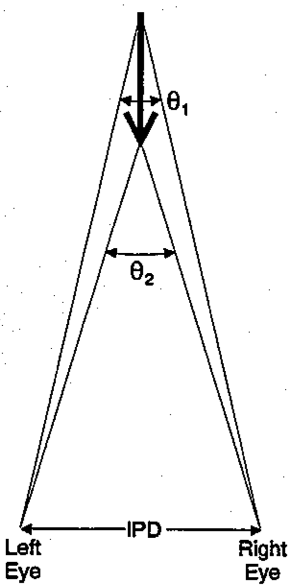


FIGURE 5.14 Perceiving depth.

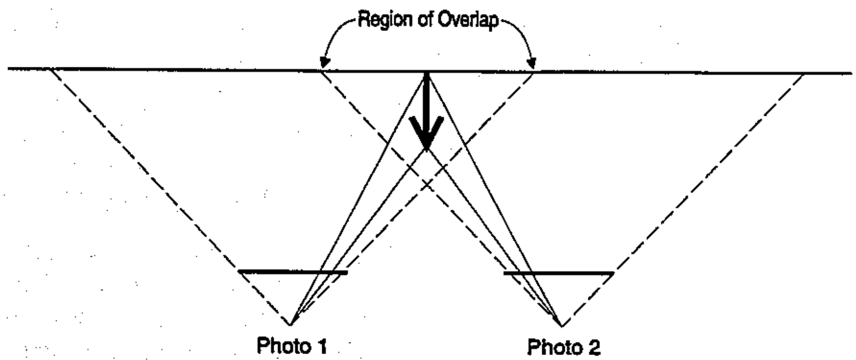


FIGURE 5.15 Camera locations for taking photographs that can later be viewed stereoscopically.

USE OF THE STEREOSCOPE

Your laboratory kit contains a pocket stereoscope. It folds into a flat 14 x 8 cm vinyl case. It may be easier for you to follow the some of the discussion that follows if you examine it as you work through the remainder of this lesson.

When you remove the pocket stereoscope from its case, you will notice a movable metal bar attached to the outside of each of two metal housings. Each of the housings contains a glass lens. The bars serve as legs that lock into place when they are opened. When the legs are open, you will notice that the distance between the lenses can be adjusted by moving the housings along a bar that shows a scale marked from 50 to 70 mm. This scale is used for setting the width of the lens so that it corresponds with your IPD. When you are ready to look through the stereoscope, it is placed so that you can look through the two lenses simultaneously. The numbers on the scale should be facing you.

You likely have seen a 3-D movie at some time. When a 3-D movie is shown, two projectors are used. Each projector shows images taken simultaneously from separate cameras, offset some distance from each other. If you have ever seen a 3-D movie, you will likely remember the funny-looking glasses with tinted lenses you were given to wear. When you looked through the glasses, you probably were able to see images clearly in three dimensions. When you took the glasses off, images on the movie screen appeared quite fuzzy and out of focus. This occurred because the glasses were tinted in such a way that your left eye was able to see images from the left projector only and your right eye images from the right projector. This enabled each of your eyes to focus independently on the appropriate images. These images were then merged into a single image within the optic centre of your brain, producing a sense of depth.

The pocket stereoscope works in much the same fashion as the tinted glasses. However, the stereoscope uses ground glass lenses to keep the line of sight of each eye parallel while enabling you to focus your left eye on the left print and your right eye on the right print, in spite of the natural tendency of unaided eyes to converge when they focus. The stereoscope also magnifies the image so that you can make out greater detail.

Stereoscopic Viewing of Aerial Photographs

We suggest that you do not try to view a pair of photographs stereoscopically the first time you read this section. If you have never viewed aerial photographs stereoscopically before, it is easiest to start your stereoscopic viewing with the stereograms included with the assignment. Only after you are sure that you can see stereoscopically, and know what an object viewed from above looks like in 3-D, should you try to view your photographs stereoscopically.

It is essential to align aerial photographs properly if you are to be able to view them stereoscopically. The top of the photographs in your laboratory kit can be distinguished by the **identification information** which should be in the upper right hand corner when the photograph is properly aligned. The identification information on your photographs will look like '30BC79046 No 00*xy*' where '*xy*' can have values between 36 and 39 inclusive. The '30BC79' portion of the code tells you that the photographs were the 30th set flown in British Columbia (BC) in 1979 (79). The '046' portion of the code refers to the **flight line** number. Flight line is the name given to the path an aircraft follows along a single bearing. The flight line for your photographs followed a bearing of 270 degrees (i.e., due west). This means that the top of the photograph represents true north when the photograph is properly aligned. (Note that most, but not all, flight lines follow an east or west bearing. If you contract your own aerial photographs you can specify the direction of the flight line in any direction you wish.) The 'No 00*xy*' portion of the code refers to the **photo number**. This refers to the position of the photograph within a flight line. Your photographs are numbered from 0036 to 0039 inclusive. The numbers increase from east to west as the photographs progress across the UBC Endowment Lands and campus area.

To view a pair of photographs stereoscopically, place two consecutively numbered photographs on a flat surface with the top edges furthest away from you. Make sure the photograph with the smaller number is on the right. Try to work in a location where the lighting is good and glare is minimal. Initially, ensure that both photographs receive roughly equivalent light.

Place the edge of one of the photographs (it doesn't matter which one) beneath the edge of the other photograph. Slide the photographs together, positioned so that their lines-of-flight coincide. (You will be marking the line-of-flight on each of your photographs in the assignment.) Pick an object that you can see clearly in both photographs and adjust the photographs so that the images of this object are separated by a distance roughly equivalent to your IPD. Place the stereoscope on top of the images so that each of the lenses is centered over the object you wish to view. When you look into the stereoscope, you will probably see two separate images of the object you are viewing. Move the photographs slightly while still looking through the stereoscope until the two images of the object appear to merge into one. When this occurs, you should suddenly see the image appear three-dimensional. The effect is quite startling at first; if you are in some doubt as to whether you are seeing stereoscopically, then you most likely are not.

Many people have some difficulty seeing aerial photographs in stereo at first, so if you can't do it right away do not get discouraged. If you have difficulty, the best thing to do is to take a break and try again later. A small proportion of the population cannot see stereoscopically. This is generally due to damage in one eye which leads to the other eye being considerably more dominant. However, everyone has one eye that is more dominant than the other. Sometimes you can improve your ability to see stereoscopically by strengthening the light source on the side of your weak eye.

You can easily find out which of your eyes is dominant by trying an experiment. Hold your arm out in front of you with your thumb extended vertically. With both eyes open, position your arm so that your thumb is blocking an object across the room from where you are sitting. Close one of your eyes and see whether your thumb appears to shift. Open that eye and close your other eye. The eye you are looking through when your thumb does *not* appear to shift is your dominant eye.

Preparing Aerial Photographs for Long-Term Stereoscopic Viewing

In this final section of this lesson, we describe how to prepare your photographs for comfortable, long-term stereoscopic viewing. The procedure has been divided into steps in order to make it easier to follow. You will be asked to prepare each of your photographs as part of the assignment. Be careful how you do this because it can affect greatly the accuracy of measurements you will be making in later assignments. Also, you will be making marks on the photographs that cannot be removed. It is possible to ruin the photograph for any future use.

step 1

• ***Remove Margins from the Photographs:***
This is not an essential step, but removing the margins widens the area of easily obtained stereoscopic coverage. The margins on your photographs consist of an outer white frame and an inner black frame. One of the margins contains some instrument readings that inform you of the date of the photograph and the time of day among other things. You may wish to make note of whatever readings you consider useful before removing the margins. (None of this information is required for your assignments.)

The margins can be removed easily with a pair of scissors. Be sure to cut each of the sides following a straight line, and cut right up to the edge of the print. Note that the fiducial marks and the photo identification are within the edges of the print and should not be removed.

step 2 • ***Locate and Mark Principal Points:***
You will recall from earlier in this lesson that principal points are located in the exact centre of the photographs and that the fiducial marks are used for locating this point. The procedure that should be followed is quite simple. Lay a straight edge across the photograph from the centre of one fiducial mark to the centre of the opposite mark. Use the push pin included with your photographs to scratch a thin line a few cm long in the emulsion in the vicinity of the middle of the photograph. Repeat this for the other pair of fiducial marks. The point at which your scratched lines cross is the principal point. Use your pin to make a small hole through the photograph at that point. On the back of the photograph circle this hole in ink and label it 'PP*xy*', where '*xy*' is the photo number. Use the wax pencil provided to draw a line over the scratch marks and then lightly erase this line. This should remove wax from the emulsion, but leave wax where the emulsion has been scratched. You will be left with a thin coloured cross with a pin hole in its centre that marks the exact location of the principal point.

step 3 • ***Locate and Mark Conjugate Principal Points:***
Recall that conjugate principal points are images of principal points from adjacent photographs in a specific photograph. In order to mark these points accurately, you need to take advantage of stereoscopic viewing and the fact that principal points can be precisely located using the fiducial marks. We will assume in the following description that the principal points are already marked on all the photographs.

Set up an overlapping pair of photographs so that you are viewing the principal point of the left photograph stereoscopically. (This will require that the right photograph be partially overtop the left photograph.) You should see the cross marking the principal point of the left photograph quite clearly although it is only marked on that photograph. Place a pin in your right hand and attempt to place the point of the pin directly onto the image of the principal point in the right photograph while you are viewing that point stereoscopically. Look up from the stereoscope and visually check that the point of the pin appears to be in the right place. Try this a number of times until you become confident in your ability to do this accurately. Mark the conjugate principal point by making a hole in the photograph with your pin at the appropriate location. On the back of the photograph, circle the hole and label it 'CPP*xy*' where '*xy*' is the photograph for which this point is the principal point. On the front of the photograph, scratch a 1 cm x 1 cm cross in the emulsion with the conjugate principal point in the centre of the cross. Draw a line over the cross with your wax pencil and then lightly erase this line.

The same procedure is followed for marking the conjugate principal point on the left photograph except that the principal point on the right photograph is viewed stereoscopically and the pin for marking the location of the conjugate principal point on the left photograph should be held in your left hand.

step 4 • ***Mark the Line-of-Flight:***
The principal point and conjugate principal points on an aerial photograph represent the only points over which you can be sure that the aircraft flew. If yaw is present, the line-of-flight will not pass exactly through the fiducial marks. Since yaw may not be constant between adjacent photographs, the line-

of-flight may not be exactly straight. In recognition of this, the line-of-flight on any photograph is drawn as two straight line segments. Each segment begins at the principal point and proceeds through one of the conjugate principal points out to the edge of the photograph. These segments should be drawn on the front of the photograph in ink so that the line is both thin and permanent.

step 5 • ***Measure Average Photo Base Distances:***
A photo base distance is the distance between a principal point and a conjugate principal point on a given photograph. The average photo base distance is the average of this distance on two adjacent photographs. For example, the average photo base distance for photos 37 and 38 would be the average of the distances between PP37 and CPP38 on photo 37 and PP38 and CPP37 on photo 38. These distances may not be the same because of measurement error and the impact of changing terrain.

Photo base distances should be measured with a ruler to the nearest millimetre and recorded on the back of every photograph. Average photo base distances should be calculated and recorded on the backs of odd numbered photographs. For example, the average photo base distances for photos 36 and 37, and for photos 37 and 38, should both be recorded on the back of photo 37.

step 6 • ***Tape Photographs Down for Long-Term Stereoscopic Viewing:***
You will notice that aerial photographs are easily knocked out of alignment for stereoscopic viewing if they are not secured in place. This is particularly bothersome and can contribute error when making the measurements required to calculate the height of objects. (Several ways of calculating heights will be covered in the next lesson.) This can be avoided by taping the photographs to your viewing surface. If you are careful when you initially align the photographs, the whole of the overlapping region should be in proper alignment for stereoscopic viewing with no further adjustments required.

Begin by taping the outside corners of one of a stereo pair of photographs onto your viewing surface. Slide the other photograph towards the taped photograph so that their lines-of-flight coincide. It does not matter whether the inside (adjacent) edge of the free photograph is slid below or above the taped photograph. Stop when you can see the central portion of the matching edge in stereo. Move the stereoscope to the top of the photograph and make whatever fine adjustments you need to make to the free photograph until you can see this portion in stereo. Repeat this for the bottom of the photograph. Continue to make fine adjustments up and down the edge until the whole side appears to be properly aligned. At this point, tape down the outside edges of the free photograph. Check to ensure that you can see stereoscopically anywhere in the region of overlap by reversing the position of the free edges of the photographs. (In other words, lift the free edge of the photograph presently on the bottom and place it on top of the free edge of the other photograph.) If you can see stereoscopically all along this edge, then you have the photographs in perfect alignment. If you cannot see this portion stereoscopically, then you should remove the tape from the photographs and begin the procedure anew.

REVIEW/SELF-STUDY QUESTIONS

Do these questions before you go on to complete the Graded Assignment. These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. Differentiate between the terms *photogrammetry* and *photo interpretation.*
2. List several reasons why aerial photographs are widely used in forestry and other fields.
3. What are some of the general uses of aerial photography in forestry?
4. Define the following terms: *nadir, ground datum,* and *line-of-flight.*
5. Distinguish roll, pitch, and yaw from each other. What is the impact of each of these on the relationship between the nadir and the principal point?
6. Differentiate between *principal point* and *conjugate principal point.*
7. What are fiducial marks?
8. What is a truly vertical aerial photograph?
9. What is tilt? How is it controlled operationally?
10. What is the primary cause of yaw, and what is its impact?
11. What is the electromagnetic spectrum and what range of wavelengths does it encompass?
12. What is atmospheric attenuation? Why is it important?
13. What are pass bands? Why are they important?
14. What is near infrared radiation?
15. What happens when EMR falls on a surface?
16. What is a spectral response curve?
17. Why can spectral signatures not be used to identify objects from aerial photographs?
18. What is the standard observer curve?
19. What determines how something appears in an aerial photograph?
20. What are the purposes of the film/filter component of the photographic system?
21. What are the purposes of the camera/lens component of the photographic system?
22. Describe the factors which determine how much light reaches a film.
23. What is relative aperture?
24. Does a smaller *f*/stop represent more or less light than a larger one? Why?

25. What is scale? What causes it to change across an aerial photograph?

26. What is meant by a nominal scale on an aerial photograph and how is it determined?

27. What is it that causes our sense of depth? Why doesn't everyone have the same sense of depth?

28. What is the principle upon which the stereoscope operates?

29. How can you determine the top of an aerial photograph?

30. Why might an individual not be able to see stereoscopically?

31. What is average photo base distance? Why might the corresponding photo base distances on a stereo pair of photographs be different from each other?

LESSON 6 PRINCIPLES OF PHOTOGRAMMETRY

INTRODUCTION

LESSON OVERVIEW

This lesson covers the basic geometric theory behind the techniques for measuring the heights of objects on vertical aerial photographs. Techniques will be developed for both single photographs and stereo pairs.

LESSON OBJECTIVES

After completing this lesson and the assignment, you should be able:

1. to understand the theory behind several different methods of measuring the height of objects from vertical aerial photographs;
2. to determine the heights of objects on single vertical aerial photographs from measurements of displacement or shadow length;
3. to determine the heights of objects in the area of overlap on a pair of stereo photos from measurements of parallax difference.

LESSON READINGS

Much of the material in this lesson is covered by Avery and Berlin on pages 49 to 90. Some of this material is covered in Avery and Burkhart, pages 263–265.

LESSON ASSIGNMENT

When you have completed this lesson, answer the self-study questions at the end. You should then complete Graded Assignment #5 and mail it to your tutor by the date indicated on your course schedule. Be sure to include a pink assignment cover sheet.

DISPLACEMENT DUE TO RELIEF

You learned in the last lesson that scale varies in an aerial photograph as a consequence of changes in topography. At this point it is worthwhile to ask the question, "What is the consequence of the fact that scale varies on aerial photographs?" In the previous lesson we outlined several uses of aerial photographs that involve making measurements, so you know that the answer to this question is not, "Measurements cannot be taken on vertical aerial photographs." A partial answer to this question is, "Aerial photographs are not maps and therefore should not be expected to yield exact measurements of distance or bearings." However, there are ways of getting around this problem as you will see in Lesson 7. A more positive answer is, "We can exploit the fact that scale varies to determine the heights and relative elevations of objects." The methods that can be used to do this are the major thrust of this lesson.

DERIVATION OF A FORMULA FOR DISPLACEMENT DUE TO RELIEF

We will derive this formula using the geometric theory of similar triangles. Following the derivation is much easier if you use the diagram in Figure 6.1. Four triangles will be used: (1) Lxy; (2) LXY; (3) Lxz; and (4) LX'Z. The following distances have been given special labels:

- L to x is called f (the focal length of the lens);
- L to X is called H (the flying height above ground datum);
- x to y is called a (the photo distance between the base of the object and the principle point);
- x to z is called a' (the photo distance between the top of the object and the principle point);
- X to Y and X' to Z are called A (the ground distance the object is a way from the nadir);
- Z to Y is called h (the height of the top of the object above ground datum).

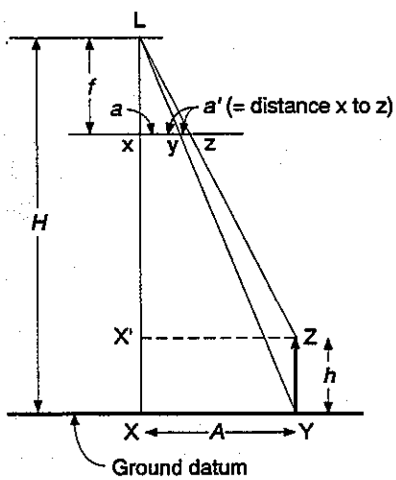


FIGURE 6.1 Diagram for use in deriving the formula for displacement due to relief.

Triangle Lxy is similar to triangle LXY. From this similarity, it follows that:

$$\frac{f}{H} = \frac{a}{A}$$

(Recall that we derived this relationship in the previous lesson when we were exploring scale.) Cross-multiplying yields:

$$aH = Af. \qquad \text{[Call this } \textit{Result 1.}\text{]}$$

Triangle *Lxz* is similar to triangle LX'Z. From this similarity, it follows that:

$$\frac{f}{H-h} = \frac{a'}{A}$$

Cross-multiplying yields:

$$a'(H-h) = Af \qquad \text{[Call this } \textit{Result 2.}\text{]}$$

Result 1 and *Result 2* provide two different equalities for the product Af. It follows that:

$$aH = a'(H-h)$$

$$aH = a'H - a'h$$

$$a'h = a'H - aH$$

$$a'h = (a' - a)H.$$

Now we will make some judicious substitutions. We will let $(a' - a)$ equal d (d is short for displacement; $(a' - a)$ is the displacement of the top of the object from its base on the photograph). Next, we will let a' equal r (r is short for radial distance; a' is the radial distance of the top of the object from the principal point on the photograph). After these substitutions, the formula becomes:

$$rh = dH$$

This can be rearranged to yield:

$$d = \frac{rh}{H}$$

This is the formula for displacement due to relief in a single aerial photograph. It is the first of three formulae that we will derive this lesson. The following observations can be made from the relationships expressed in this formula:

- d is directly proportional to r.
  This means the farther the top of an object is away from the principal point, the greater the displacement. Hence, an object of a given height will be displaced more if it is close to the edge of the photo than if it is close to the principal point. If the object is found exactly at the principal point (i.e., $r = 0$), then there will be no displacement.
- d is directly proportional to h.
  This means that the taller the object, the greater the displacement. We will exploit this relationship for calculating the height of objects from a single aerial photograph later in this lesson.
- d is inversely proportional to H.
  This means that the lower the flying height, the greater the displacement.
- d is not directly related to scale.
  This is useful because you are seldom sure of the exact scale at any point on a photograph.
- d is radial from the principal point.
  You should be able to see this quite clearly from your photographs.

As an example of how you can apply some of this material, look at the following problem taken from an old examination.

Suppose picture A (Figure 6.2) is taken with a camera whose lens has focal length f at an altitude of H above ground datum. Suppose picture B is taken with a camera whose lens has focal length $2f$ at an altitude of $2H$ above the same ground datum. Assume both pictures are vertical and that exposure conditions (illumination, nadir, aircraft heading, cross wind, etc.) are identical. Is picture A the same as picture B? Explain.

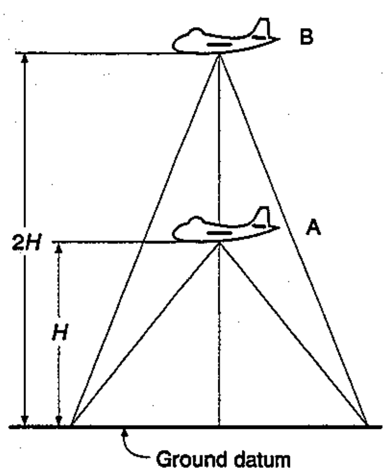


FIGURE 6.2 Diagram for the sample problem.

Picture A differs from picture B. (No, not only because picture B has an airplane in it!) The explanation does not lie with scale. The scale for picture A is f/H. The scale for picture B is $2f/2H$ which equals the scale of picture A. The difference between the pictures is due to displacement caused by relief. The displacement of an object r units from the principal point in picture A is rh/H. The displacement of an object r units from the principal point in picture B is $rh/2H$ which equals 1/2 of the displacement in picture A.

It is possible to obtain a wide range of displacements for a single scale. This can be done by changing the focal length of the lens an amount proportional to any change in flying height so that f/H remains constant. Hence, whether you can accurately measure the heights of trees using aerial photographs depends upon flying height, not scale.

Measurement of Heights of Objects Using a Single Photograph

Heights of objects are generally measured using overlapping pairs of stereo aerial photographs. The reason for this is a matter of measurement precision rather than theoretical necessity. As you will see, there are two ways that heights can be calculated using only a single photograph.

method 1

• ***Using Displacement Due to Relief:***
The formula that we derived in the previous section for displacement due to relief can be rewritten to yield a formula for calculating height:

$$h = \frac{dH}{r}$$

As an example, we will use this formula to calculate the height of the clock tower in photo #38. We measured the displacement (d) of the clock tower as 1.5 mm. The top of the tower is approximately 96 mm from the principal point (r). We will assume that the flying height of the aircraft above the base of the clock tower is 3000 m. (In order to calculate the height of an object we need to move the ground datum to the base of that object.) Given these measurements and assumptions, the height of the clock tower is:

$$h = \frac{dH}{r} = \frac{1.5 \times 3000}{96} = 47 \text{ m}$$

How precise is this calculation? One way to determine this is to look at the reliability of the measurements. Let's assume that we measured the distances on the photograph to the nearest 0.5 mm. Therefore d can range between 1.25 and 1.75 mm and r can range between 95.75 and 96.25 mm. To create a worst case scenario, we need to look at the calculated value of h with d and r at opposite extremes. When $d = 1.75$ and $r = 95.75$, height becomes:

$$h = \frac{1.75 \times 3000}{95.75} = 54.6 \text{ m}$$

When $d = 1.25$ and $r = 96.25$, height becomes:

$$h = \frac{1.25 \times 3000}{96.25} = 38.7 \text{ m}$$

Hence, the height of the clock tower could realistically be within the range of 38.7 m to 54.6 m. This is not very precise although we would do well to measure distances with a standard ruler to the nearest 0.5 mm.

The impact of flying height on the precision of height measurements can be easily seen through a further illustration. Let's see what will happen to the precision of the clock tower measurement if the flying height is halved. We will assume that the focal length is also halved so that the scale remains constant. If we do this, it means that r won't change from the example above. Obviously, the height of the clock tower (h) will not change either. Since $d = rh/H$, halving H must mean that d will double to 3 mm. The precision with which we can make measurements on the photograph remains ± 0.25 m. Hence, at one extreme $d = 3.25$ mm, $r = 95.75$ mm, and $H = 1500$ m. Using these figures, the estimated height is:

$$h = \frac{d \times H}{P} = \frac{3.25 \times 1500}{95.75} = 50.85 \text{ m}$$

At the other extreme, $d = 2.75$ mm, $r = 96.25$ mm, and $H = 1500$ m. Using these figures, the estimated height is:

$$h = \frac{d \times H}{P} = \frac{2.75 \times 1500}{96.25} = 42.75 \text{ m}$$

Thus, the impact of halving the flying height is to narrow the range of our estimate to between 42.75 and 50.85 m. This is obviously an improvement over the range we obtained with a flying height of 3000 m.

To show the impact of r on this measurement, we could go through a similar illustration. Say we chose to halve r. If H is held constant, this would mean d would also be halved. We will leave the calculations for you as an exercise. (You do not have to submit the results.) You should find that the precision of the height measurement will be worse. Another way of saying this is, "The farther the object you are measuring is away from the principal point, the more precise your measurement." You should keep this in mind when choosing a photograph on which to make measurements.

method 2

• ***Using Shadow Length:***
If the shadow cast by an object is visible on a vertical aerial photograph, the length of the shadow can be used to estimate the height of the object. The first step is to convert shadow length on the photograph (s) to shadow length on the ground (S). This is done by multiplying s by the inverse of scale (H/f):

$$S = \frac{s \times H}{f}$$

If s and f are measured in millimetres and H is in metres, then the units of S will also be metres. The next step is to use trigonometry to figure out the height of the object (h) from S and the elevation angle of the sun (Θ) (Figure 6.3). If h is vertical, then:

$$\tan(\Theta) = \frac{h}{S}$$

$$h = S \times \tan(\Theta)$$

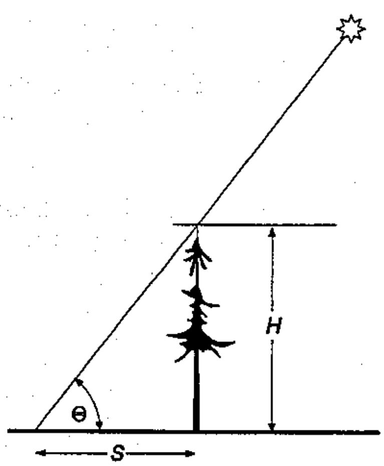


FIGURE 6.3 Determining the height of an object using shadow length.

The elevation angle of the sun (θ) can be determined in two ways:

- from knowledge of the height of an object on the photograph with a clearly visible shadow. In this case, it is a simple matter to solve the equation given above for θ as: $\theta = \arctan(h'/S')$ where h' is the known height and S' is the shadow length of the object on the ground.
- from tables of sun angles (known as a solar ephemeris). In order to find the sun angle, you need to know: (1) the date of the photography; (2) the time of the photography to the nearest hour; and (3) the latitude and longitude of the location. Items (1) and (2) can be obtained from information printed on the margin of photograph.

We will not go through any examples because shadow length is a very rough means of determining height of objects, and is seldom used in practice. Some of the reasons why shadow length does not provide reliable heights include:

- the formula requires knowledge of scale at a point;
- the formula assumes the object is perpendicular. This will not be the case if the object is leaning or if the ground is not flat;
- the shadow of the object you are interested in may be wholly or partly obscured;
- the object may not throw a shadow from the very top;
- the ground level may be obscured by brush, snow, and so on.

Other Measurements from a Single Photograph

Other measurements may sometimes be taken on single aerial photographs, including distance between two points, area, and the bearing between two points. These measurements should be applied with caution because they will be exact only if the ground is perfectly flat. In practice, they frequently are used to provide rough approximations. Even under these conditions, care should be taken if the terrain is quite uneven. There is little point in using expensive instruments to make these measurements since errors due to terrain differences will likely mask any errors introduced by inexpensive instruments.

Distance measurements on a single photograph are generally made using either a standard ruler or a rolling wheel planimeter. A rolling wheel planimeter is composed of three basic parts: (1) a weighted arm of fixed length (polar arm); (2) a trace arm hinged on the unweighted end of the polar arm; (3) a rolling wheel which rests on the measurement surface attached to a vernier scale. To use it to measure distance, the trace arm is moved along the line which you are measuring. The distance is read off the appropriate vernier scale in mm.

Measurements of area are sometimes made from aerial photographs to estimate the area of some delineated shape (e.g., timber type). Two types of instruments are commonly used: dot grids and rolling wheel planimeters.

Dot grids are comprised of a transparent sheet covered with a fixed number of dots per cm^2. The dot grid is placed at random over the shape of interest and the number of dots that fall into that shape counted. The total count is then divided by the number of dots per cm^2 to determine the area of the shape in cm^2. This can be repeated several times in order to calculate an average area. The area in cm^2 on the photograph is then converted into a ground area (usually in m^2 or ha) using the approximate scale of the photograph.

To determine area using a planimeter, the pointer is run around the boundaries of the area in a clockwise direction. The area in cm^2 is read directly from the appropriate vernier scale. As with the dot grid, this works best if it is repeated a few times and the results averaged. The photograph area is then converted to a ground area using the approximate scale.

The exact bearing between any two points can be found only if the photographs are truly vertical and the terrain is flat. The one exception to the requirement of flat terrain is if the bearings are from the principal point. This is because displacement due to relief is always radial from the principal point. This fact is exploited when maps are constructed from aerial photographs, as you will see in Lesson 7.

STEREOSCOPY

Stereoscopy is the science or art that deals with stereoscopic (3-dimensional) effects and the methods by which these effects are produced. In Lesson 5, we explained how people see depth. In this section, we look at the maximum distance at which an average person can discern depth. We also explain the phenomena which cause you to have an exaggerated sense of depth when you view an overlapping pair of aerial photographs.

RADIUS OF STEREOSCOPIC PERCEPTION

The **radius of stereoscopic perception** refers to the maximum distance at which we can perceive a sense of depth. We will calculate this radius first for an average individual with unaided eyes. Recall that the average individual can sense angles of convergence on the order of 0.00556 degrees (Θ) and that the average interpupilary distance (IPD) is 65 mm. In order to determine the radius of stereoscopic perception (R), we must calculate the distance at which the angle of convergence of the eyes is less than 0.00556 degrees. It may help you to follow the calculations if you refer to Figure 6.4.

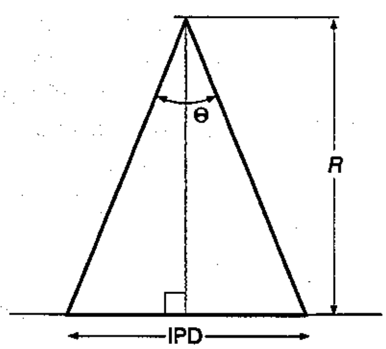


FIGURE 6.4 Maximum radius of stereoscopic perception.

$$\tan(\Theta/2) = \frac{\text{IPD}/2}{R} = \frac{\text{IPD}}{2R}$$

$$R = \frac{\text{IPD}}{2\tan(\Theta/2)} = \frac{65\text{ mm}}{2\tan(0.00556/2)} \cong 670\text{ m}$$

Thus, the maximum distance that the "normal" human can discern depth with the unaided eye is approximately 670 m.

Radius of stereoscopic perception can be increased in two ways. One way is to increase the virtual base line (i.e., the IPD in Figure 6.4). There is a proportional relationship between the length of the base line and the radius of stereoscopic perception so that any increase in the length of the base line immediately causes a corresponding increase in the radius. The other way is magnification. For instance, a magnification of 10 times essentially makes objects appear 10 times closer to the observer. This would cause the radius of stereoscopic perception to be increased 10 times.

A common instrument that employs both these strategies is binoculars. Most binoculars increase IPD by two times and also magnify by some power as well. A set of binoculars which have 7 power magnification increases depth perception over unaided eyes by 2 x 7 which equals 14. Therefore, the radius of stereoscopic perception would become 14 x 670 which equals 9380 m.

VERTICAL EXAGGERATION

There is no absolute vertical scale in vertical aerial photographs since different people have different IPD's and hence see depth differently. However, there is a sense of vertical exaggeration. In other words, objects appear taller than they should given the horizontal scale. This is related to the **base/height ratio** (B/H) where B is the ground base distance between principal points on adjacent photographs and H is the flying height above ground datum. The angle of convergence increases for a given object as B increases or H decreases. Therefore, vertical exaggeration increases with an increase in the base/height ratio.

Photo base distance is more or less fixed for any nominal scale of photography by the size of the print and the amount of overlap required. We will assume for the following calculations that the photo base distance for standard 23 cm x 23 cm prints is 95 mm. (This will vary up to several millimetres depending upon the terrain.) For a nominal scale of 1:10,000, 95 mm on the photo represents 950 m on the ground. This increases IPD by a factor of 950 ÷ 0.065 which equals 14,615. On 1:10,000 nominal scale photographs, there is a reduction (the opposite of magnification) of 10,000 from the ground conditions. Therefore, sense of depth is increased by a factor of 14,615 ÷ 10,000 which approximately equals 1.5. Thus, you see a vertical exaggeration of about 1.5 times when you view your photographs stereoscopically.

Would this change if you looked at 1:20,000 photographs? Not if the prints are the same size as the 1:10,000 photographs and the horizontal overlap is the same. You can easily verify this by repeating the calculations that we showed you for the 1:10,000 photographs. The photo base distance will still be 95 mm. At a nominal scale of 1:20,000, this represents 1900 m on the ground. This increases IPD by a factor of 29,231. The reduction due to scale is 20,000. Hence, vertical exaggeration is 29,231 ÷ 20,000 which approximately equals 1.5 again. Another way to determine this is to use the base/height ratio. Decreasing the nominal scale from 1:10,000 to 1:20,000 doubles the ground base distance. In order to decrease the scale to 1:20,000, flying height must double (assuming focal length remains the same). Hence, the base/height ratio remains the same for 1:20,000 photographs.

In order to check your understanding of this process, see if you can do the calculations for determining the vertical exaggeration for the object shown in Figure 6.5 when it is viewed using photos 1 and 3.

Answer: Vertical exaggeration is approximately 3 times.

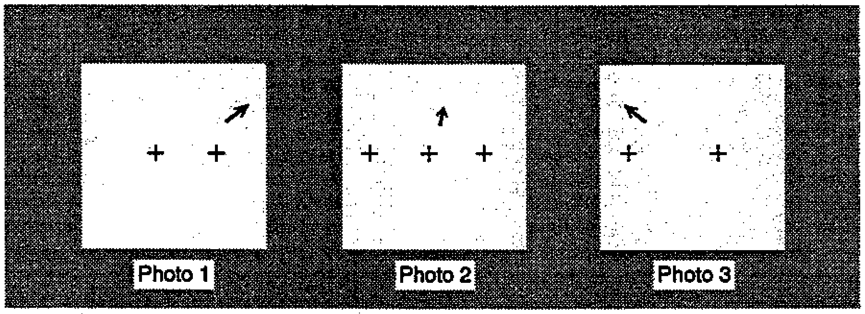


FIGURE 6.5 Location of an object on three consecutive photographs.

PARALLAX

As mentioned previously, people discern the depth of an object viewed from above using the differences in convergence of their eyes when viewing the top versus the bottom of the object. If it were possible to accurately measure the differences in convergence of your eyes when you are viewing an object stereoscopically on a pair of aerial photographs, this could be used to determine the height of the object. However, such angles are very difficult to measure so this does not provide a viable means of determining object heights.

Fortunately, there is another way to measure heights of objects on stereo pairs of photographs using linear measurements in a manner similar to that used for single photographs. This involves the notion of parallax and parallax differences, which is the subject of the remainder of this lesson.

ABSOLUTE STEREOSCOPIC PARALLAX

Look at the arrow shown on the two aerial photographs in Figure 6.6. We have exaggerated the amount of yaw on the two photographs to illustrate that the photographs are lined up along a common l-o-f, not along a line drawn parallel to the base of the photographs. The arrow on the photographs represents a vertical object on the ground. Notice that the arrow is pointing away from each of the principal points. This is because the top of the arrow is displaced radially from the principal points as a function of displacement due to relief. If an object is located between the principal point and corresponding conjugate principal point on a pair of overlapping photographs, the tops of the object in the two

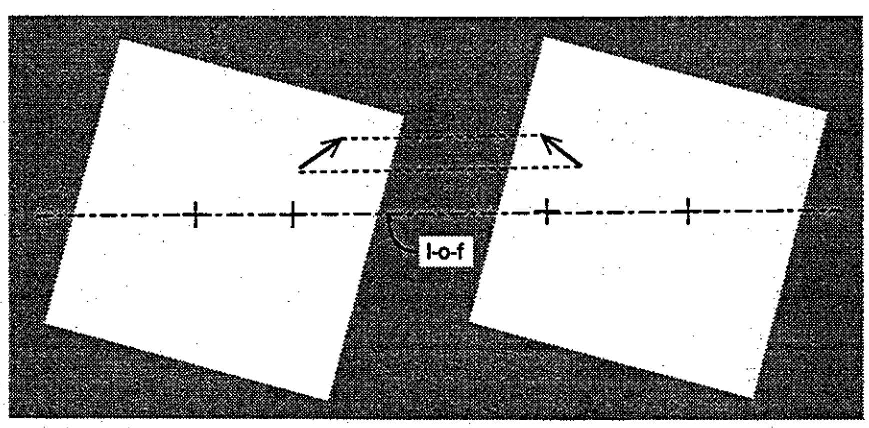


FIGURE 6.6 The appearance of a vertical object on an overlapping pair of aerial photographs.

photographs will be closer together than the bases when the photographs are lined up along a common l-o-f. This will remain true no matter what the distance is between the photographs as long as the right photograph remains on the right. Furthermore, the difference in distances between the tops of an object and the bases of the same object is related to the height of the object. The taller the object, the greater the distance.

Any point in the zone of overlap on a pair of aerial photographs will have an **absolute stereoscopic parallax** (P) associated with it. The absolute stereoscopic parallax of a point is the algebraic difference, measured parallel to the l-o-f, of the distances of the two images from their respective principal points, assuming the photographs are truly vertical and are taken from the same height above ground datum. This definition can be more easily understood by looking at Figure 6.7. In this figure, we have constructed a Cartesian coordinate system using the l-o-f as the X-axis, and a perpendicular drawn to this line through the two principal points as Y-axes. The point has coordinates (x_1,y_1) in photo 1 and (x_2,y_2) in photo 2. By definition, $P = x_1 - x_2$. It is for this reason that absolute stereoscopic parallax is sometimes called X-parallax. Note that x_2 is negative in this example because it falls to the left of the Y-axis on photo 2. In this situation, we could write P as being equal to $x_1 + x_2$. This is usually, but not always, the case as you will soon see.

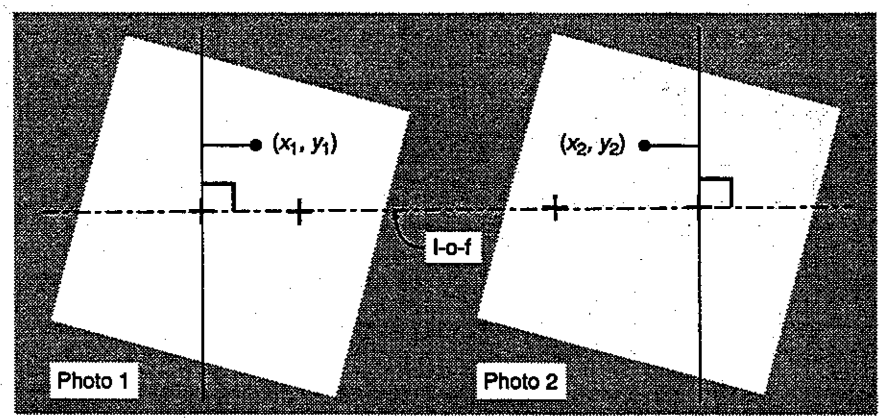


FIGURE 6.7 Locating a point on a pair of overlapping aerial photographs using a Cartesian coordinate system.

We can think of the area of overlap between a pair of aerial photographs as being comprised of three regions (Figure 6.8). For simplicity we will describe these regions only in terms of their locations on the left photograph (photo 1). Region 1 comprises a narrow strip to the left of the principal point on photo 1 out to the left edge of the zone of overlap. If a point falls into region 1, both x_1 and x_2 are negative. In terms of absolute distances, $P = x_2 - x_1$, however, the formula $P = x_1 - x_2$ still holds in terms of algebraic distances. Region 2 comprises the area between the principal point and the conjugate principal point. This is the major area of overlap for which we developed the formula in the previous paragraph. Region 3 comprises the area to the right of the conjugate principal point out to the edge of the photograph. It is the complement to region 1 in that both x_1 and x_2 are positive. In terms of absolute distances, $P = x_1 - x_2$. Again, the formula $P = x_1 - x_2$ holds in terms of algebraic differences.

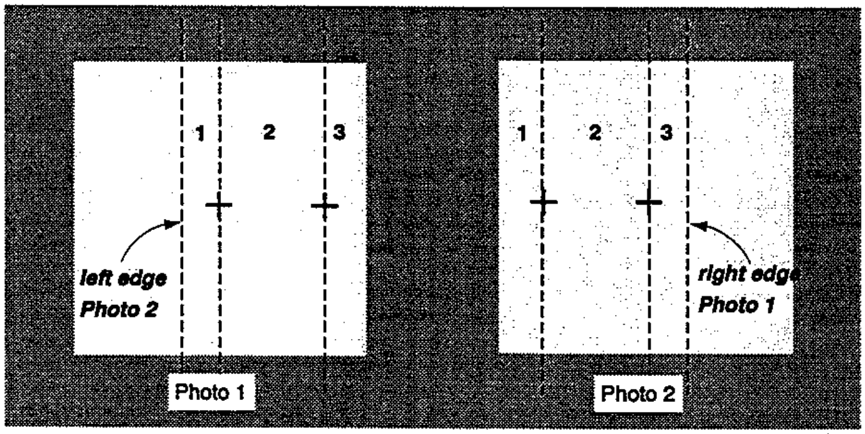


FIGURE 6.8 Regions in the area of overlap for a pair of aerial photographs.

The fact that you can measure absolute stereoscopic parallax in terms of distances rather than angles is to your advantage since distances are much easier to measure. However, measuring *P* may not be easy, nor does it provide you with a direct means of obtaining the height of the object. It turns out that you can measure the difference in the parallax between the bottom of the object and the top of the object (known as **parallax difference** or *dP*) more accurately than you can measure *P*. Furthermore, *dP* can be related directly to the height of the object. How this procedure works will be the subject of the next section.

Developing a Formula for Determining the Height of Objects

We begin this section by deriving a formula known as the **parallax theorem** that will allow the parallax at a point of known elevation with respect to the ground datum to be determined. We will then make use of this formula in deriving another formula that will allow the height of any object in the zone of overlap of two photographs to be calculated.

In order to follow the derivation of the parallax theorem, it is essential to refer to the diagram in Figure 6.9.

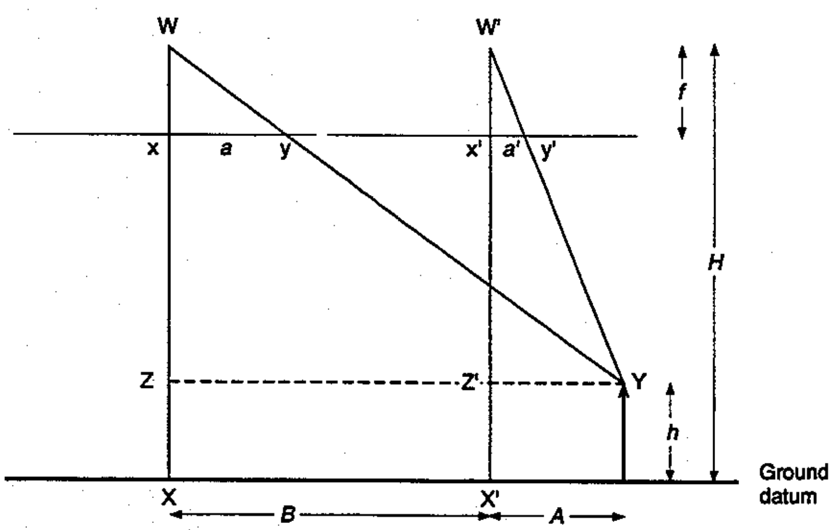


FIGURE 6.9 Diagram for developing the parallax theorem.

Triangles Wxy and WZY drawn from photo 1 are similar. From the theory of similar triangles, it can be shown that:

$$\frac{a}{f} = \frac{A+B}{(H-h)}$$

$$a = \frac{f(A+B)}{(H-h)} = \frac{fA+fB}{(H-h)}$$

Triangles W'x'y' and W'Z'Y drawn from photo 2 are also similar. From the theory of similar triangles, it can be shown that:

$$\frac{a'}{f} = \frac{A}{(H-h)}$$

$$a' = \frac{fA}{(H-h)}$$

The parallax of the point at elevation *h* above ground datum is:

$$P = a - a' = \frac{fA+fB}{(H-h)} - \frac{fA}{(H-h)} = \frac{fB}{(H-h)}$$

where *B* is the ground base distance. The following formula is known as the parallax theorem:

$$P = \frac{fB}{(H-h)}$$

In words, this formula states that the absolute stereoscopic parallax of a point at an elevation of *h* above the ground datum is equal to the focal length of the camera times the ground base distance divided by the difference between the flying height above ground datum and the elevation of the point above ground datum.

This formula could be rewritten to give us a formula for the height of any object above ground datum. We could also use the actual flying height above the base of the object (*H'*) rather than *H*. The difficulty with using this formula to solve for *h* is that it contains *B*, the ground base distance. This is rarely known exactly and estimating it from the photo base distance and the approximate scale is prone to error unless the terrain is relatively flat.

It turns out that a formula that does not contain *B* can be derived. In order to illustrate the derivation, Figure 6.9 is redrawn in a slightly different fashion as Figure 6.10. Refer to that diagram to help follow our derivation.

Let *P* be the absolute stereoscopic parallax at the base of the object.

$$P = a_1 - (-a_1') = a_1 + a_1'$$

The parallax difference is:

$$dP = P_{TOP} - P$$

$$P_{TOP} = P + dP$$

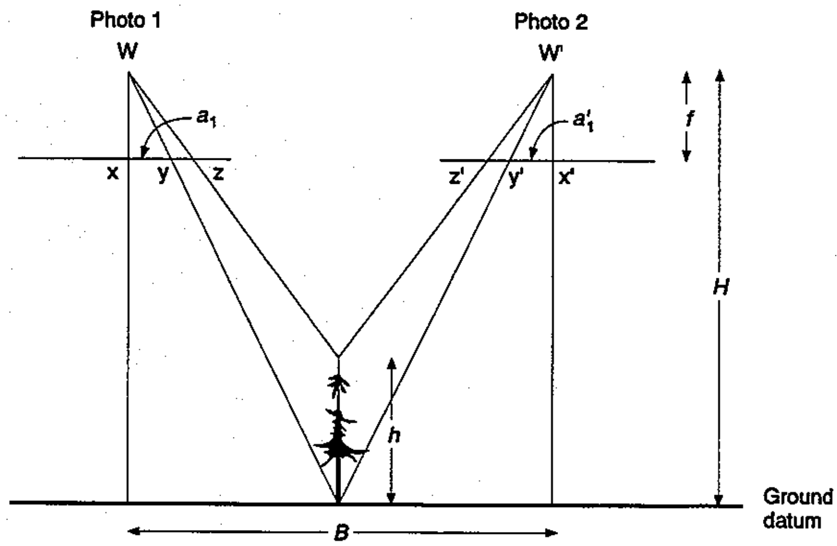


FIGURE 6.10 Diagram for deriving a formula for measuring the height of an object from an overlapping pair of photographs.

From the parallax theorem, we know:

$$P = \frac{fB}{H-h}$$

Since $h = 0$ at the base of the object if H is the flying height above the base, the parallax at the base of the object is:

$$P = \frac{fB}{H}$$

The parallax at the top of the object is:

$$P + dP = \frac{fB}{(H-h)}$$

Our objective is to solve for h as well as get rid of B. We can do that with some mathematical juggling.

$$\frac{P}{P_{TOP}} = \frac{P}{P+dP} = \frac{fB}{H} \times \frac{H-h}{fB} = \frac{H-h}{H}$$

$$H - h = \frac{HP}{P+dP}$$

$$h = H - \frac{HP}{P+dP} = \frac{HP + HdP - HP}{P+dP} = \frac{HdP}{P+dP}$$

The final formula is:

$$h = \frac{HdP}{P+dP}$$

In order for this formula to give an exact answer for height, the symbols must be defined in the following fashion:

h = height of the object above its base;

H = flying height above the base of the object (i.e., ground datum is moved so that it coincides with the base of the object);

P = absolute stereoscopic parallax at the base of the object;

dP = parallax difference between the top and the base of the object.

USING THE FORMULA

1) ***Determining P:***
Absolute stereoscopic parallax at the base of an object is difficult to measure exactly, even with expensive equipment. Fortunately, P can be easily approximated. Recall that:

$$P = \frac{fB}{H - h}$$

If h is considered to be the elevation difference between some point and the ground datum, this formula provides the absolute stereoscopic parallax of that point. Recall also that scale (S) at any point is:

$$\frac{f}{H - h}$$

where h is the elevation distance between the point and the ground datum. B is equal to the distance between the principal points on the ground. Since $S \times B$ is equal to the average photo base distance (b), b can be substituted for P. This substitution will be exact if:

- the principal points have the same elevation;
- the base of the object is at this elevation.

If these conditions are "almost" true, the average photo base distance can be used to approximate P. Frequently this is done in practice.

Let's look at this in a slightly different way by considering the absolute stereoscopic parallax in the two adjacent photographs shown in Figure 6.11.

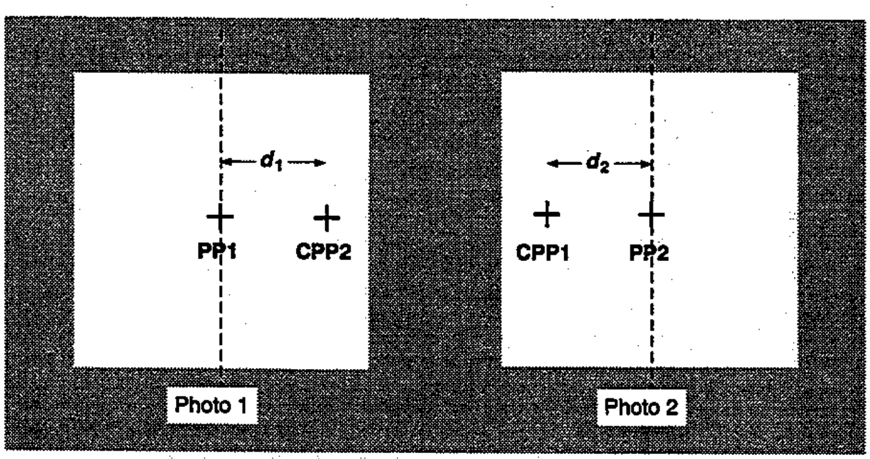


FIGURE 6.11 Parallax at the principal points of two adjacent aerial photographs.

P at PP_1 is $0 - (-d_2)$ which equals d_2. P at PP_2 is $d_1 - 0$ which equals d_1. If h' at PP_1 equals h' at PP_2, then P at PP_1 equals P at PP_2 (i.e., $d_1 = d_2$). Otherwise these two differences are not equal. Recall that average photo base distance is calculated as:

$$b = \frac{d_1 + d_2}{2}$$

Thus, b may be considered an "average" value for P at elevations between the h' at PP_1 and the h' at PP_2.

2) ***Measuring dP:***
Parallax difference can be measured quite exactly by exploiting the ability of the human eye to discern depth. An instrument designed for this purpose, known as a **floating mark stereometer** (also sometimes called **height finder** or **parallax bar**), is included in your kit. This instrument uses the same principles as more expensive electronic equipment, but cannot achieve the same degree of accuracy.

The stereometer is approximately 18 cm long and 4 cm wide and is stored in a vinyl case. It consists of a metal base, two plastic tabs each with a small etched dot, and a metal wheel containing a scale. There are two grooves, one on each end of the underside of the base. The grooves are designed to attach to the legs of the stereoscope. The stereometer should be attached so that the plastic tabs face away from you when you look through the lens of the stereoscope. One of the plastic tabs is fixed; the other may be moved along the base by turning the wheel. The scale on the wheel consists of 100 numbers. Each rotation of the wheel moves the plastic tab 1.0 mm, so the distance between each of the numbers on the wheel represents 0.01 mm.

Before you use the stereometer, it is important that you securely fasten the pair of photographs you are using to a flat surface in a comfortable position for long-term stereo viewing. (Recall that this procedure was covered in the previous lesson.) Once this is done, place the stereoscope, with the stereometer attached, in a position where you can see the object of interest stereoscopically. Adjust the stereoscope and the plastic tabs on the stereometer so that it appears that the dots cover the object top on each of the photographs. Look through the stereoscope and fine-tune the stereometer so that the dots appear to merge. At this point you should be seeing the object stereoscopically and a single dot floating at apparently the same elevation as the top of the object. If you turn the wheel on the stereometer backwards and forwards slowly, the dot should appear to rise and fall. Reset the merged dot at the top of the object and take a reading. Move the wheel until it appears that the object is floating just at the ground surface. Take another reading. The top reading minus the bottom reading will equal the parallax difference. You will have an opportunity to practice using the stereometer in the assignment.

Another inexpensive device that can be used for measuring parallax difference is the **parallax wedge.** The parallax wedge consists of two graduated lines on some transparent material. One line has no numbers, coarse graduations, and is vertical. The other line has numbers, finer graduations and slopes away from the vertical line. The numbers on this line represent the distance the lines are apart at that point. The lines can be thought of as a series of parallax bars, set at a reading different from the previous one by a constant amount. An example of a parallax bar with distances in inches is reproduced in Figure 6.12.

The first step to operating the parallax wedge is to align the photographs for stereoscopic viewing and tape them into place. View the object of interest stereoscopically and place the parallax wedge over top of the object. The two lines on the parallax bar should fuse together along one section of their length. Because the lines are different distances apart, the fused portion will appear to float in space. Move the wedge about the photographs until the fused portion of the line appears to cut across the top of the object. Record the width at that point. Move the wedge again until the fused line seems to intersect the ground at the base of the object. Record the width at that point as well. The difference between the reading at the top of the object and the reading at the base is the parallax difference.

It takes quite a bit of practice to become proficient in using the floating mark stereometer and the parallax wedge. An operator can become consistent to within 0.05 and 0.10 mm when measuring parallax difference. In contrast, precision electronic equipment can measure parallax difference to about 0.0005 mm. However, this equipment is both expensive and bulky.

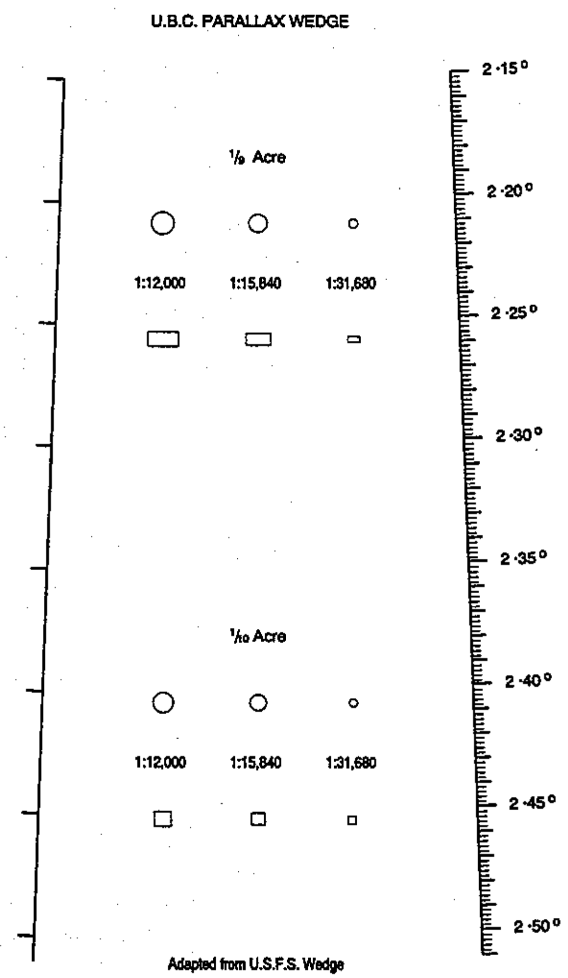


Figure 6.12 An example of a parallax wedge.

The parallax wedge is cheaper than the stereometer and does not contain any mechanical parts that could be knocked out of adjustment. The stereometer is easier to learn how to use and its measurements are repeatable without the possibility of a previous reading biasing the results.

3) ***Errors in Calculating Height:***
Errors can accumulate from several sources. The sources (in decreasing order of importance from the standpoint of this course) are:
- Measuring *dP* (on the order of 10% with your level of skill and instrumentation);
- Determining precise height (H') of the aircraft above the base of the object (on the order of 1%);
- Using average photo base distance (b) to approximate the absolute stereoscopic parallax (P) at the base of the object (on the order of 1%);
- Motion of objects in the wind (possibly on the order of 15% in a reasonably strong wind).

A number of precautions can be taken to minimize the impact of these errors. The precision of your measurements of *dP* will improve with practice. Usually a photo interpreter must have between six and eight years of experience before undertaking contour mapping. Also, using expensive electronic positioning instruments greatly improves the precision. Precise knowledge of flying height above the base of an object is particularly important on large scale aerial photographs. It is possible to use an instrument called a radar altimeter to obtain a much more precise height above ground than a standard barometric altimeter. Absolute stereoscopic parallax at the base of the object can be precisely measured with expensive positioning equipment, but these measurements are time consuming. Often parallax is measured only a few times per photograph if the terrain is approximately level. Errors arising from the motion of objects can be minimized by taking a stereo pair of photographs simultaneously. This is frequently done for large-scale aerial photographs using cameras mounted either on the wings of a fixed-wing aircraft or on a boom suspended below a helicopter. Generally, aerial photographs are not flown on a windy day.

REVIEW/SELF-STUDY QUESTIONS

Do these questions before you go on to complete the Graded Assignment. These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. What is displacement due to relief and how can it be calculated?
2. What are the relations between displacement and:
   (i) radial distance of an object from the principal point;
   (ii) height of the object;
   (iii) flying height; and
   (iv) scale?
3. What is the difficulty with determining the height of an object from a single photograph using displacement due to relief?
4. Explain how shadow lengths could be used to determine the heights of objects.
5. What are some of the practical difficulties with using shadow length?
6. Besides measurement of heights of objects, what other measurements may be made off single photographs? What is the major factor affecting the accuracy of these measurements?
7. Identify two instruments for measuring the area of a region on an aerial photograph and briefly describe how each of the instruments is used.
8. What is stereoscopy?
9. What is the radius of stereoscopic perception?
10. How can the radius of stereoscopic perception be increased?
11. Why is there no absolute vertical scale in aerial photographs?
12. What is the base/height ratio and what does it govern?
13. Differentiate between absolute stereoscopic parallax and parallax difference.
14. Why is it awkward to use the parallax theorem to determine the height of objects in many cases?
15. What is the relationship between the parallax at the base of an object and the object's elevation above ground datum?
16. How is absolute stereoscopic parallax at the base of an object normally approximated? Under what conditions will this approximation be exact?
17. What two simple instruments can be used to measure parallax difference? What are the advantages of each?
18. What are the major sources of error in calculating the heights of objects from a stereo pair of aerial photographs? What can be done to minimize the impact of each of these error sources?

LESSON 7 — MAPPING SYSTEMS AND MAPPING FROM AERIAL PHOTOGRAMMETRY

INTRODUCTION

LESSON OVERVIEW

In this lesson, an overview of map projection techniques and mapping systems is first presented to provide background for a detailed discussion of planimetric mapping from aerial photographs. The theory behind planimetric mapping is covered, as are ground control points, photo control points, and flight planning. Radial line triangulation is presented as a simple means of transferring data from an aerial photo to a map. Topographic mapping from aerial photographs is covered briefly.

LESSON OBJECTIVES

After completing this lesson and the assignment, you should be able:

1. to differentiate among several of the major map projection techniques;
2. to explain the principles that apply to the construction of planimetric and topographic maps from aerial photographs;
3. to construct a simple planimetric map using radial line triangulation.

LESSON READINGS

Material covered in this lesson may be found in Avery and Berlin, pages 91–140. Some of this material is covered in Avery and Burkhart, pages 258–262.

LESSON ASSIGNMENT

When you have completed this lesson, answer the self-study questions at the end. You should then complete Graded Assignment #6 and mail it to your tutor by the date indicated on your course schedule. Be sure to include a pink assignment cover sheet.

OVERVIEW OF MAPPING THEORY

There are two basic types of maps:

• **planimetric maps** which show horizontal (i.e., ground) position of details of the Earth's surface. These details can include shorelines, rivers and streams, roads, buildings, civil boundaries, and so on.

• **topographic maps** which show elevation (relief) in addition to some details of the Earth's surface. Relief can be shown using contour lines, shading, and so on.

Aerial photographs play an important role in making both kinds of maps.

Despite the fact that a vertical aerial photograph is not a precise record of distances (i.e., scale varies because of displacement due to relief), it is a precise record of angles as measured from the principal point with respect to the line of flight. As you learned last lesson, this is because all displacement is radial from the principal point. As you will see, it is this factor that is exploited in map making.

We will begin this lesson by providing an overview of general mapping theory. This is not intended to be exhaustive, but it will provide you with some background. We will then look at planimetric mapping in some detail. We will conclude the lesson by briefly discussing the use of aerial photographs in topographic mapping.

MAP PROJECTION PROCEDURES

It is difficult to make maps of larger areas (e.g., British Columbia, Canada, the world) because the Earth is approximately spherical and a sphere can not be flattened onto a plane (map) without distortion. A map-maker would like to achieve true distance, true direction, true shape and true area on a map. There is no way to achieve all four of these properties. A number of different projection procedures have been developed; we will briefly describe the basics of some of these below. However, we will first define some terms that will help you follow the descriptions more easily.

definitions

A number of technical terms are frequently encountered when reading about maps. Some of these terms may already be familiar to you, but we expect that a few will be new.

Parallels: lines of equal latitude. These lines run east-west and are the same distance apart anywhere on the globe.

Meridians: lines of equal longitude. These lines run north-south on the globe. They are farthest apart at the equator and converge to a single point at the poles.

Rhumb Lines: lines of true bearing.

Great Circle: formed by the intersection of the Earth's sphere with a plane passing through the centre of the Earth. The arc of a great circle is the shortest distance between any two points on the Earth's surface. Arcs of great circles do not have a constant bearing unless they happen to run in a north-south direction or coincide with the equator.

Equivalent Projections: The area of a portion of the Earth's surface is the same on a map as it is on a globe of the same scale, but the shape is not the same.

Conformal Projections: The shape on the map is the same as on a globe, but the area is different.

Orthographic projection

Orthographic projection is the simplest of all projection procedures because it does not account at all for the curvature of the Earth's surface. Orthographic projections are produced by projecting detail on the Earth's surface onto a plane that is tangent to the Earth (i.e., just touches the Earth) at the centre of the projection (Figure 7.1a). This type of projection creates distortions everywhere except at the centre of the projection. The amount of distortion increases with distance from the centre. In other words, the distance between equally spaced points on the globe becomes greater as you move away from the centre. The magnitude of the distortion associated with this technique generally limits its application to small areas of just a few square km.

Lambert Conformal projection

The **Lambert conformal projection** technique involves projecting detail from the Earth's surface onto a cone with its apex centred over one of the poles (Figure 7.1b). The cone is positioned so that the lower portion intersects the globe along two standard parallels in such a manner that two-thirds of the north-south portion to be mapped lies between these parallels. The remaining area to be mapped is equally divided so as to lie one-sixth north and south of the standard parallels. Details are projected onto the cone, and the cone is cut and flattened to produce a map. This technique produces meridians that are straight lines radiating from the poles and parallels that are arcs of concentric circles. There is less distortion in an east-west than in a north-south direction.

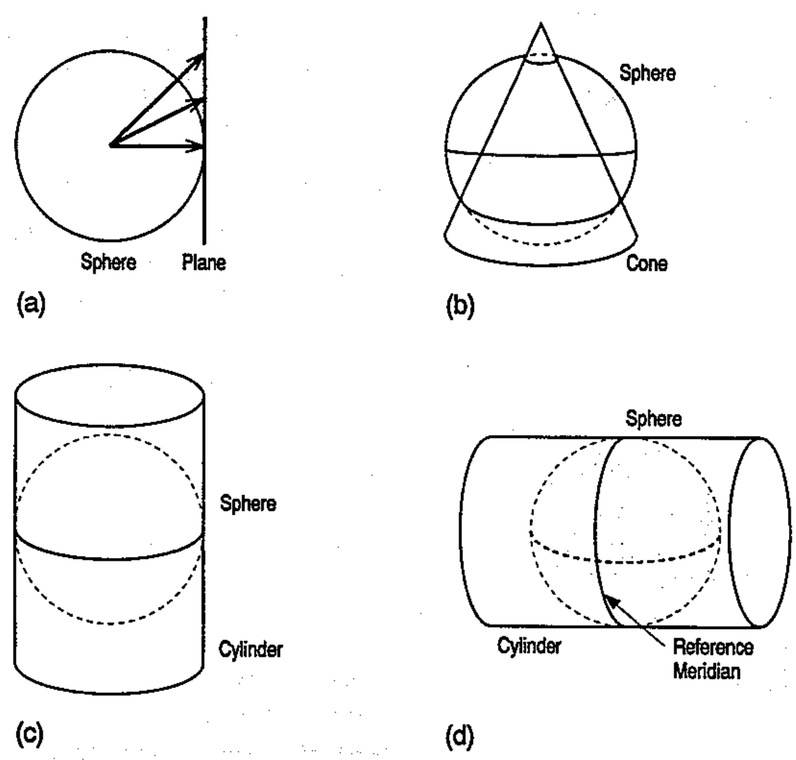


FIGURE 7.1 Map projection variations: (a) orthoganal; (b) Lambert conformal; (c) Mercator; (d) transverse Mercator.

Mercator projection

With the **Mercator projection** technique, points on the Earth's surface are projected onto a vertical cylinder with a north-south axis that is tangent to the Earth at the equator (Figure 7.1c). Points on the Earth's surface are projected outwards until they intersect the cylinder. The cylinder is then cut and flattened into a map. Mercator projection is very good for regions around the equator. Distortion in area increases as distance from the equator increases.

You often see Mercator projection used for maps of the complete globe. Meridians appear as straight vertical lines which means that they must be spaced apart everywhere except at the equator. The amount of distortion increases greatly towards the poles. At 60 degrees north or south the amount of distortion is approximately two times; at 80 degrees the distortion is approximately six times. At the poles, the distortion approaches infinity.

One of the main advantages of Mercator projection is that a straight line drawn in any direction is a rhumb line, which is useful for producing navigation maps. No other projection technique produces maps with this property.

transverse Mercator projection

Transverse Mercator projection is similar to Mercator projection except that the cylinder is turned on its side (Figure 7.1d). It touches the Earth at the north and south poles, and along two complementary meridians. The identity of the meridians is determined by the positioning of the cylinder with respect to the globe. Transverse Mercator projection is very good for points close to the reference meridians, but distortion increases in an east-west direction as distance from the reference meridian increases.

universasl transverse Mercator (UTM) projection

Universal transverse Mercator (UTM) projection shows generally less distortion than regular transverse Mercator projections. The difficulty with east-west distortion in transverse Mercator projection was solved in the same manner as time zones. The globe was separated into 60 equal zones, each with its own reference meridian. Since there are 360 degrees of longitude, each zone covers 360 ÷ 60 which equals 6 degrees of longitude. Hence, no point in an east-west direction is more than 3 degrees from a reference meridian. Since this approach allows the complete globe to be mapped with an acceptable amount of distortion, it was called "universal."

A reference grid commonly called the UTM grid, which follows this projection system, has been established world-wide. Both the U.S. and Canada use UTM projection for national mapping and UTM projection is also used by all the provincial governments. However, some of the U.S. state governments use other techniques for producing state maps.

NTS Map Coordinate System

Map coordinate systems are ways of identifying individual maps within a map series. These systems work in much the same way as reference systems in a library that allow you to easily find the shelf location of a book from among the thousands of books in the library. Many different map coordinate systems exist world-wide. One such system, the **National Topographic System** (NTS), used to locate maps produced by the federal government, is described below. This system is superimposed on top of UTM projections which determine the spatial locations of the mapped items. An example of the complete numbering sequence and the corresponding scales is given in Figure 7.2.

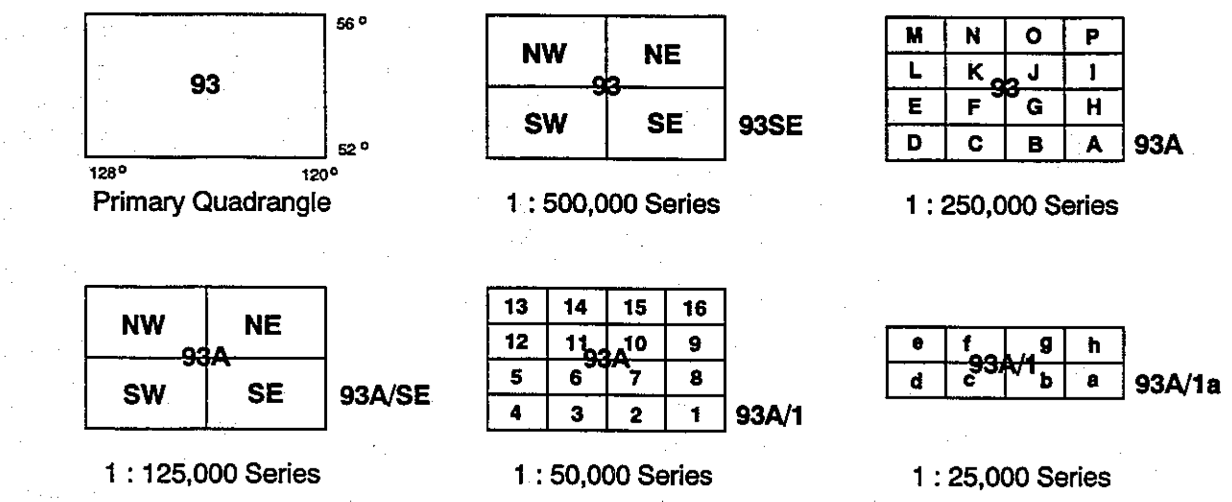


FIGURE 7.2 The National Topographic System (NTS) of identifying maps.

The largest unit in the system is the primary quadrangle. This is an area that covers 8 degrees of longitude by 4 degrees of latitude. Primary quadrangles are identified by a number (e.g., 93). No map sheets are produced for an area of this size.

The smallest scale maps in the system are the 1:500,000 series. Each of these maps covers one-quarter of a primary quadrangle. The location of a map sheet in this series within a primary quadrangle is designated by the primary quadrangle number and its orientation within the primary quadrangle (e.g., 93SE for the southeast quarter within primary quadrangle 93.)

The next smallest scale is the 1:250,000 series. Each of these maps covers one-sixteenth of a primary quadrangle. The location of a map sheet in this series within a primary quadrangle is designated by a capitalized alphabetic letter. Map A begins in the lower right corner. Letters proceed from right to left, and left to right in alternating rows up the primary quadrangle to finish with Map P in the upper right corner. Maps are identified by the primary quadrangle number and the map letter (e.g., 93A for the map in the lower right corner of primary quadrangle 93).

The 1:125,000 series represent quarters of the 1:250,000 series. As with the 1:500,000 series, the quarters are identified by their orientation. Maps are identified by the 1:250,000 series number followed by a slash and the orientation (e.g., 93A/SE for the map covering the southeast quarter of map 93A).

The next largest series is the 1:50,000 series. Like the 1:125,000 series maps, these maps represent a portion of the 1:250,000 series. However, in this case the 1:50,000 maps each cover one-sixteenth of the smaller scale maps. The maps are identified by the 1:250,000 series number, followed by a slash and then a number between 1 and 16 inclusive. The numbers progress from *1* in the lower right corner to *16* in the upper right corner following the same pattern as the letters did in the 1:250,000 series (e.g., 93A/1 refers to the map in the lower right corner of the 1:250,000 series map 93A).

The largest scale maps produced are the 1:25,000 series. Each map in this series covers one-eighth of the 1:50,000 series maps which is divided into two rows of four. Each of the eight sections is designated by a lower case letter running consecutively from *a* in the lower right corner to *h* in the upper right corner. Maps in this series are designated by the 1:50,000 series number followed by the appropriate lower case letter (e.g., 93A/1a refers to the map in the lower right corner of the 1:50,000 map 93A/1).

PLANIMETRIC MAPPING

FINDING THE RELATIVE POSITION OF A POINT

Consider the point C shown on the two photographs in Figure 7.3. The only information regarding this point that can be obtained from the photographs are the two angles formed between that point and the lines of flight on the two photographs. These angles are identified as α and β in the figure. You are unable to locate point C on a map with only this information. Additional information that is necessary includes:

- the distance between PP1 and PP2. This can be thought of as **scale.**
- the orientation of the triangle which can be formed between PP1, PP2, and point C. This can be thought of as **rotation**. Another way of thinking of this is the relationship between point C and north.
- the actual position in space of either PP1 or PP2. This can be thought of as **translation.**

This information is provided by the map maker through **ground control points** (GCP's).

Ground control points are carefully located positions on the ground and on the map that show latitude and longitude (horizontal control) and elevation above mean sea level (vertical control). Each ground control point provides two pieces of information: the latitude and the longitude of the point. Hence, the minimum number of ground control points that are required on a given map is two.

This means that if you have a blank piece of paper on which two ground control points are located, and you know the exact location of these points on your aerial photographs, you can produce a planimetric map. The location of the first ground control point can be thought of as fixing the exact location in space (i.e.,

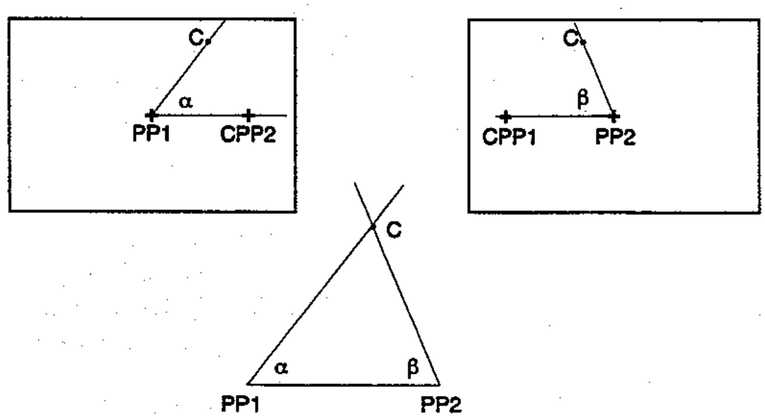


FIGURE 7.3 Information concerning a point found on two adjacent aerial photographs.

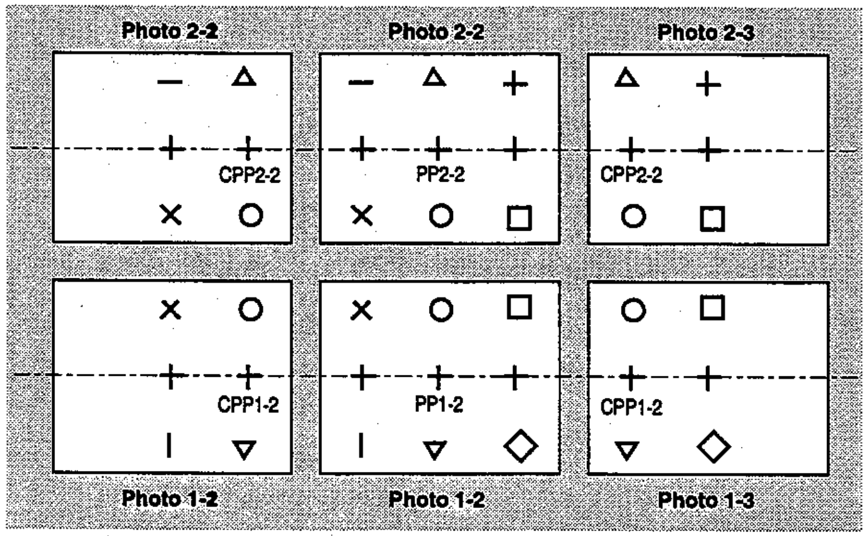


FIGURE 7.4 Location of photo control points.

translation). The location of the second ground control point establishes the distance between these points on the map (i.e., scale) and the orientation of the map (i.e., rotation).

We will return to ground control points shortly. However, it is first necessary to address **photo control points** (PCP's). These points (sometimes called 'pass points' or 'wing points') are well-defined stationary features easily pinpointed on overlapping photographs. They are used to help link photographs with no ground control points (both along and between lines-of-flight) to photographs already located on the map. The strategy of locating these points reflects their purpose. Six photo control points are located on each photograph approximately a photo base distance above and below the principal point, and each of the conjugate principal points. This location allows the points to be seen on a maximum of six photographs in a series: three on a given line of flight, and three on the line of flight immediately above or below this line of flight (Figure 7.4).

Photo control points are marked on each of the photographs on which they are found. The points are best located while viewing a pair of photographs stereoscopically. The procedure to follow is much the same as we described for locating conjugate principal points in Lesson 5. The exact location of the photo control points should be marked with a pin hole and a small coloured cross scratched in the emulsion.

NUMBER AND LOCATION OF GROUND CONTROL POINTS

Two ground control points are the absolute minimum required. In order to be useful, each of these points must appear on at least two of the photographs that cover the area to be mapped. The more ground control points located in the mapping area, the more accurate the map is likely to be. However, ground control points are expensive to establish. (They usually require a special ground survey unless you are fortunate to already have ground monuments in place.) Locating too many of these points increases the cost of making the map an inappropriate amount. The optimal number of points cannot be stated in general terms; it depends upon the desired accuracy of the map and the skill of the map makers.

The positioning of the ground control points also affects map accuracy. It turns out that control using photo control points is better within a line of flight than between lines of flight. This is due principally to the greater overlap within a line of flight. Thus, ground control points are most effective if they are spread over the map in such a way that they help link photographs from adjacent lines of flight. The usual point of maximum error on a map is the farthest point from a ground control point. Hence, it is also a good idea to spread the ground control points over the area to be mapped. Note that practical considerations do not always make this possible. For example, if a road is located along one edge of an area to be mapped and the remainder of the area is not easily accessible on foot, it may be more cost efficient to locate the ground control points along the road. Another consideration is the fact that ground control points must be accurately located on the aerial photographs. If the photographs have not been flown when the ground control points are being located, the points can be marked on the ground with targets (frequently large white crosses several metres in size) that will be visible on the photographs. This requires that the ground control points be located in an area where the ground is visible from the air (i.e., not under a dense canopy of trees).

Transferring Details from Photographs to Maps

The procedure used to transfer details from photographs to maps is called **radial line triangulation.** This technique is easy to do, but it may be difficult for you to follow at first. You will have a chance to practice in the accompanying assignment. We suggest you read through the description that follows several times before trying the assignment. You should also proceed slowly and carefully. Small errors have a tendency to accumulate and can become quite substantial even on a map as small as the one you will be making.

There are a number of ways of performing radial line triangulation. The technique we describe makes use of mylar sheets. This approach to transferring details is both inexpensive and flexible enough to handle a variety of map scales. Other techniques and equipment are described in Avery and Berlin, pages 123–130.

Our description of radial line triangulation using mylar sheets is divided into two parts. This first part comprises the steps involved in transferring the location of the principal points from the photographs to the base map:

1. Accurately locate and mark the ground control points on all the photographs in which they appear.

2. Accurately locate photo control points on all the photographs.

3. Tape a photograph containing at least two ground control points to your table. (In practice you can work with a photograph with only one ground control point, but it makes the process more complicated. One of your photographs has three ground control points so you will be able to follow this description. Once you understand the process, you will likely be able to imagine how you could transfer detail if none of your photographs has more than one ground control point.)

4. Centre a mylar sheet over top of that photograph and tape it down to your table as well. Write the photo number at the top of the sheet.

5. Poke a hole through the mylar sheet at the principal point.

6. Draw the line-of-flight on the mylar sheet. (*Note:* the mylar sheets are easier to work with later if you change the colour of ink you are using between steps.)

7. Draw a radial line from the principal point through each of the ground control points. When you are making a map approximately the same scale as the photographs (which is what you will be doing in the assignment), it is not necessary to draw the complete radial line. Rather, only a segment of the radial line extending a few centimetres on either side of the point on the photograph needs to be drawn. This will help keep the mylar sheets from getting too cluttered in the vicinity of the principal point.

8. Draw a segment of a radial line from the principal point through each of the photo control points.

9. Remove the mylar sheet and the photograph.

10. Repeat steps (3) through (9) for each of the other photographs.

11. Tape the base map onto the table.

12. Place the mylar sheet containing the most ground control points onto the base map in such a way that the radial lines drawn through the ground control points on the mylar sheet intersect the ground control points on the base map.

13. Place a mylar sheet prepared from an adjacent photograph onto the base map. Orient the two mylar sheets in such a way that their lines-of-flight coincide and the lines through the ground control points intersect at the ground control points marked on the base map.

14. Carefully tape these mylar sheets into position. Be careful not to tape directly to the base map if you can help it because you would like to remove the mylar sheets without damaging the base map.

15. Fit the other mylar sheets into position one at a time. Use the line-of-flight, ground control points, and photo control points to orient the sheets. Tape each sheet into place when it is properly located.

16. Poke a hole with a pin through the mylar sheet onto the base map to mark the locations of the principal points on the base map.

17. Remove the mylar sheets from the base map.

18. Mark the location of the principal points on the base map with small crosses. Label each of these points for future reference.

The second part of radial line triangulation is concerned with transferring object locations and shapes from aerial photographs onto base maps. The same mylar sheets that were used for transferring the principal points are used to transfer these details. The technique is similar to what we have previously described for transferring the principal points. The procedure for transferring a single point is summarized below. Note that objects are considered to be collections of single points. A rectangular building may be correctly located on a map by locating just the four corners. Objects with more complex shapes can be located exactly from a few points and the remainder of the object can then be sketched onto the map.

The steps for transferring a single point are:

1. Tape a photograph containing the point onto a flat table.
2. Tape the appropriate mylar sheet over top of the photograph.
3. Draw a radial line segment from the principal point through the point.
4. Remove the mylar sheet and photograph.
5. Repeat steps (1) through (4) for all other photographs that contain the point.
6. Place the mylar sheets into their proper positions on the base map.
7. The location of the point on the base map is determined by the intersection of the line segments drawn through that point. Mark this point by poking a small hole through to the base map using a pin.

Transferring Details from a Map to a Photograph

Transferring detail from aerial photographs to maps is much more common than transferring detail from maps to aerial photographs, but there are some occasions when the latter can be useful. The procedure is similar in concept to radial line triangulation but works in the opposite direction. In the steps that follow, we assume that mylars have already been prepared and that the principal points of the relevant photographs are located on the base map.

Transferring a point from a base map to a stereoscopic pair of aerial photographs is accomplished by following these steps:

1. Tape the base map to a flat surface.
2. Tape the mylar sheets that cover the point of interest into their proper locations on the base map.
3. On each of these mylar sheets, draw a radial line from the principal point through the position of the point of interest on the base map.
4. Remove the mylar sheets from the base map and tape them onto their corresponding aerial photographs.
5. Set up an adjacent pair of these photographs for stereoscopic viewing.
6. View the photographs stereoscopically. The location of the point of interest on the aerial photographs is found where the lines drawn on the mylar sheets appear to cross.
7. Mark this point on each of the photographs by poking a hole through the mylar sheets and underlying photograph using a pin.

LOCAL ADJUSTMENT OF SCALE

Recall from Lesson 6 that an aerial photograph is not a map because scale varies everywhere as a function of relief. However, it is possible to optically remove displacement due to relief on a photograph to produce an image that is a true representation of ground position. The resulting image is called an **orthophoto.** Orthophotos can be used as detailed planimetric maps if line maps are not available for an area, and are especially useful for the creation of **base maps.** Base maps are maps that contain little detailed information besides the locations of lakes, rivers, and roads. These maps serve as templates for more detailed maps that may be developed (e.g., forest cover maps, ecological maps, soil maps) for an area.

TOPOGRAPHIC MAPPING

A detailed description of topographic mapping and some of the instruments involved is beyond the scope of this course. The text by Avery and Berlin provides very little detail on this subject (pages 130 to 133), but there is a brief description of some of the instruments involved.

Topographic mapping requires ground control points to provide vertical as well as horizontal control. Maps are produced using electronic devices attached to plotters. Vertical positioning is determined using a "floating dot" similar in principle to what you used to determine heights of objects in Lesson 6.

REVIEW/SELF-STUDY QUESTIONS

Do these questions before you go on to complete the Graded Assignment. These questions are of value to check your understanding of the material before progressing to the next lesson, as well as later review for the final examination. *Do not submit answers to the tutor.*

1. Differentiate between planimetric and topographic maps.
2. What causes difficulties when making maps of larger areas of the Earth's surface?
3. Differentiate between: (1) parallels and meridians; (2) equivalent and conformal projections.
4. What is a: (1) rhumb line; (2) great circle?
5. Briefly describe the following projection procedures:
   (i) orthographic projection;
   (ii) Lambert conformal projection;
   (iii) Mercator projection;
   (iv) transverse Mercator projection.
6. How did the term *universal transverse Mercator projection* arise?
7. What is a map coordinate system?
8. Outline the main stages of the National Topographic System (NTS).
9. What information besides the angles formed between a point and the principal point on two adjacent photographs is required to accurately locate that point on a map? What names are given to this information?
10. What are ground control points?
11. What are photo control points? Where should they be located on a photograph? Why?
12. How should ground control points be positioned on an area to be mapped to maximize their efficiency?
13. Explain (in your own words) how radial line triangulation works for transferring point locations from aerial photographs to a map.
14. How are object locations transferred from aerial photographs to maps?
15. Explain (in your own words) how to transfer a point from a map to a stereo pair of aerial photographs.
16. What is an orthophoto?
17. How is vertical control maintained in topographic mapping?

LESSON 8 — APPLICATIONS OF PHOTOGRAMMETRY AND PHOTO INTERPRETATION IN FORESTRY

INTRODUCTION

LESSON OVERVIEW

This lesson addresses some of the forestry applications of photogrammetry and photo interpretation mentioned in Lesson 5. Topics presented include forest cover typing, the basics of species identification, photo volume equations, and applications in forest inventories. Non-photographic imaging systems and geographic information systems are also covered briefly.

LESSON OBJECTIVES

Following completion of this lesson, you will be able:

1. to explain how photo-interpretation and photogrammetry can be used in forest cover typing, tree species identification, photo volume equations, and forest inventories;
2. to outline the strengths and weaknesses of non-photographic imaging systems relative to photographic systems;
3. to identify some of the forestry uses of particular non-photographic systems;
4. to explain the rudiments of geographic information systems.

LESSON READINGS

Material relevant to this lesson may be found in the text by Avery and Berlin pages 141–249 and 323–354. Some of this material is covered in Avery and Burkhart, pages 265–270.

LESSON ASSIGNMENT

When you have completed this lesson, answer the self-study questions at the end. There is no graded assignment for this lesson.

FOREST COVER TYPING

This lesson is intended to provide you with some exposure to present and future applications of photogrammetry, photo interpretation, and related technologies. Coverage is by no means exhaustive, but it should provide you with sufficient background to allow you to pursue additional sources in these areas on your own should you so wish.

Recall that photo interpretation was defined in Lesson 5 as the identification of objects from photographs and determination of their significance or meaning. Objects differ from each other in photographs because of differences in size, tone (or colour), shape, and texture among other things. It is easy to identify differences on many occasions, but it is often a difficult task to identify why the differences exist (i.e., to determine what is changing on the ground to produce the different effects that you are seeing as differences on the photograph). It is this idea of change that is important in producing **forest cover type maps** (maps showing the location of different cover types). However, it is equally important to know what elements are present to produce the pattern visible on the photograph.

Cover typing is a term associated with subdivision of a heterogeneous forest area into more uniform subgroupings. This is often a preliminary step in conducting a forest inventory. It is usually a simple matter to differentiate uniform areas within a forest. If you look at the forest areas included on your aerial photographs you will see many distinctively different areas. However, it requires considerable experience and skill to identify what comprises these different areas and to determine which geographically separate areas may be combined into common types for inventory or management purposes.

In the following section, we will examine some of the characteristics of forest stands that cause differences in appearance on aerial photographs, and discuss how these characteristics can be quantified on 1:10,000 nominal scale photographs.

Factors that Make Stands Appear Distinctive

Forest stands can be thought of a groupings of trees that are similar in terms of certain characteristics and different from surrounding groupings of trees. Boundaries between adjacent stands may be distinct or gradual. Some stand conditions that are important for management include: 1) species composition; 2) age and stand structure; 3) density (site occupancy); and 4) site.

species composition

It is a simple matter to identify the **species composition** of a stand as being mainly deciduous (light tone, fine texture), mainly coniferous (dark tone, coarser texture), or mixed. It is difficult (although not impossible) to determine rough species composition from 1:10,000 nominal photographs. This requires considerable skill and an intimate knowledge of the area being typed. The interpreter will take advantage of land form, topography, aspect and other attributes, to provide clues as to what species may be present. As you will see later in this lesson, it is possible to identify the species of individual trees using aerial photography, but this requires larger scale photographs for accurate identification.

age and stand structure

Age is impossible to determine directly from aerial photographs, but relative size (i.e., height) may give an indication of broad age class (i.e., newly established, immature, mature, and overmature). Stand and fire history records (if available) are also used in assigning age to cover types delineated on aerial photographs.

Stand structure is a little easier to identify. If the trees are mostly of similar height then the stand is even-aged; if there are two or three distinctive canopy levels, then the stand is a two- or three-leveled stand; if there are many tree sizes present, then the stand is uneven-aged. Again, knowledge of the area and what species you might expect to find growing under different conditions is useful for classifying structure.

density

Density, or the degree of occupancy by trees of an area of ground, is easy to determine subjectively on aerial photographs, and is defined as degree of crown closure. Crown closure can also be measured accurately, particularly on large-scale photographs using fine **dot grids**. Dot grids are transparent sheets containing a specified number of dots per square centimetre. The number of dots falling on tree canopies is divided by the total number of dots in a given cover type and multiplied by 100 to provide percentage crown closure.

site

It is impossible to determine **site quality** exactly using only aerial photographs. However, knowledge of terrain type, topographic position of the stand, aspect, and other features allow subjective assessment. **Site index** (height of dominant trees within a stand at some reference age) can sometimes be determined from height measurements and knowledge of stand age obtained from ground surveys or existing records.

THE BASICS OF TREE SPECIES IDENTIFICATION

The recognition of tree species on 1:10,000 nominal scale aerial photographs is most easily done when the species grows in pure, even-aged stands. If the stands are comprised of a mixture of species, then identification of single trees becomes more difficult. Species identification is more of an art than a science, although experienced interpreters can be quite accurate. Often the interpreter is helped by knowledge of the area and analysis of the prevalent landforms. Interpreters also make frequent visits to classified stands to "ground check" their interpretation.

Tree characteristics that will help you to identify species on larger scale photographs (e.g., greater than 1:2,000) are presented in this section, with particular reference to B.C. tree species that typify these features. You will see that there may be considerable overlap of features among species. Also, many trees do not look "typical" for various reasons (e.g., crowding, open-grown, damage). The experienced interpreter is able to implicitly combine several features together when identifying species and properly identify most trees.

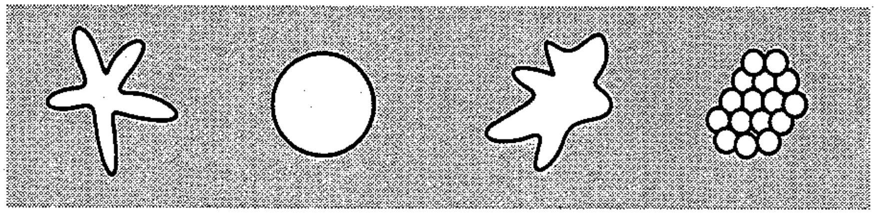
MAJOR IDENTIFICATION FEATURES

Five tree characteristics are normally examined stereoscopically to aid in identification. These include: crown boundary (outline); crown topography; crown tone or hue; branching habit; and foliage density.

Each feature is described in turn. We do not expect you to remember the features associated with any of the species, but you should know what each of the characteristics represent.

crown boundary

Crown boundary refers to the cross-sectional outline of the crown when seen from above. There are four basic patterns (Figure 8.1):

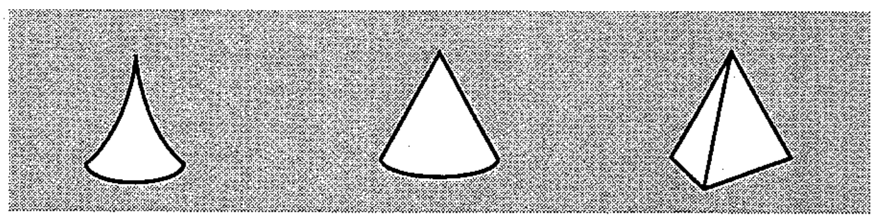


(a) Star-shaped (b) Circular (c) Irregular (d) Net patterned

FIGURE 8.1 Crown boundaries: (a) star-shaped, typical of older Douglas-fir and western white pine; (b) circular, typical of young trees of many species, including spruces, true firs, and many pines; (c) irregular, typical of some hardwoods, open-grown and damaged trees; (d) net patterned, typical of alder.

crown topography

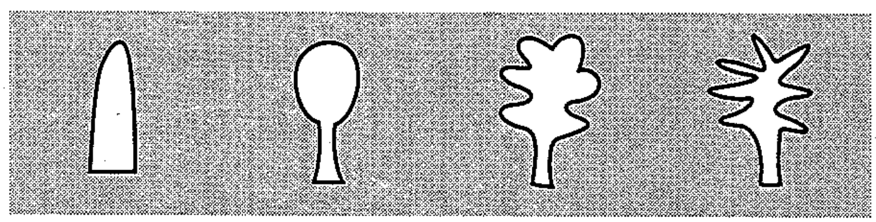
Crown topography refers to the appearance of tree crowns in aerial photographs when they are viewed vertically through a stereoscope. Tree crowns appear either concave or convex. There are three concave-shaped crowns (Figure 8.2):



(a) Spire-shaped (b) Conical (c) Pyramical

FIGURE 8.2 Concave crown topography: (a) spire-shaped, typical of true firs and young western hemlock; (b) conical, typical of western redcedar, western hemlock and Douglas-fir; (c) pyramidal, typical of western redcedar, Douglas-fir and open-grown crowns.

There are four convex-shaped crowns (Figure 8.3):



(a) Cylindrical (b) Globose (c) Billowy (d) Tufted

FIGURE 8.3 Convex crown topography: (a) cylindrical, typical of spruce and some older Douglas-fir; (b) globose, typical of lodgepole pine, ponderosa pine and aspen; (c) billowy, typical of cottonwood and maple; (d) tufted, typical of maple.

crown tone or hue

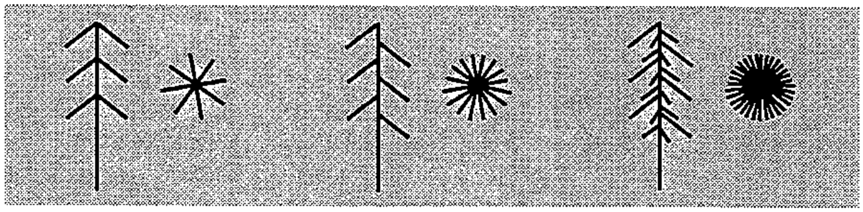
Crown tone refers to the general lightness or darkness of the crown. This is mainly pertinent to black and white photography such as you have been using for this course. **Hue** refers to the colour of the foliage, and so is pertinent only to coloured photography. Tones and hues are highly variable within a species and vary with site, age, and vigor. These characteristics are most useful when differentiating among species within a stand. The following are some general tones and associated species:

- light tones, typical of western redcedar, pines, young trees and stressed trees;

• medium tones, typical of western hemlock, western redcedar, maturing trees;
• dark tones, typical of short-needled pines, spruces, and some old-growth trees.

branching habit

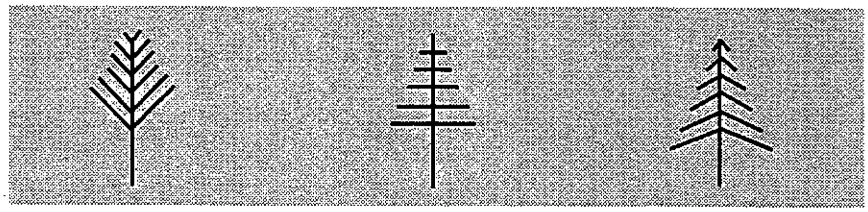
Branching habit can be divided into a number of subcategories. **Branch arrangement** refers to the initial distribution of branches along a stem (Figure 8.4).



(a) Opposite (b) Alternate (c) Alternate and spiral

FIGURE 8.4 Branch arrangement: (a) opposite which results in an apparent layering of branches, typical of pines, spruces and true firs; (b) alternate which does not produce a pronounced layering effect, typical of hemlock and western redcedar; (c) alternate and spiral which causes many branches to be visible, typical of hemlock and western redcedar.

Branch direction refers to the angle between the branches and the main stem (Figure 8.5).



(a) Ascending (b) Horizontal (c) Drooping

FIGURE 8.5 Branch direction: (a) ascending, typical of Douglas-fir; (b) horizontal, typical of pines and true firs; (c) drooping, typical of spruces, western hemlock and western redcedar.

Branch form refers to whether a branch is straight, forked, or curved. Sometimes branch form depends on tree age. For example, young western hemlock has many visible single forked branches. As the hemlock matures and foliage quantity increases, the branches begin to become fan-shaped. In mature hemlock, the branches are broadly fan-shaped. Western redcedar, on the other hand, has many visible straight branches that seldom fork. This is one of the best means of distinguishing between western hemlock and western redcedar.

foliage density and pattern

Foliage density and **pattern** refers to the arrangement and density of the foliage seen on the branches. Some examples of foliage density and pattern include:
(a) clumped in small tufts, typical of lodgepole pine;
(b) clumped in large tufts, typical of ponderosa pine;
(c) pendulous (i.e., branches with foliage hanging down making the branches visible and the crown appear thin), typical of western redcedar;
(d) striated (i.e., the foliage covers the entire branch making the branch appear "hairy"), typical of western hemlock;
(e) cigar-shaped (i.e., the foliage on the branch has a rounded, cigar-like appearance usually hiding the branch), typical of Douglas-fir and spruces.

PHOTO VOLUME EQUATIONS

In Lesson 3 we discussed volume equations that were based on ground-based measurements of particular single-tree characteristics, and briefly mentioned photo volume equations. Photo volume equations relate volume to more easily measured variables on aerial photographs. Photo volume equations can be developed to estimate both single tree volume and stand volumes. Each of these applications is described in this section.

ESTIMATES OF SINGLE TREE VOLUME

Regression techniques are used to estimate coefficients using sample data. The main advantage of photo volume equations over ground-based volume equations is lower measurement costs; once the equations are constructed ground-based measurements can be reduced or avoided entirely. The major disadvantage of photo volume equations is lower accuracy.

The poorer accuracy of photo volume equations versus ground-based volume equations is due principally to two factors: measurement errors on the photograph; and the inability to directly measure DBH on aerial photographs. Measurement errors can be reduced by careful interpreter training and the use of large-scale photographs. DBH may be estimated indirectly from other factors or DBH can be replaced by other independent variables (usually crown diameter or crown area) in volume equations.

The best variables to measure on large scale photographs for estimating volume are height and some aspect of crown dimension (usually crown diameter or crown area). These variables are useful because they can be easily and accurately measured, and they have a good relationship to volume.

You learned about measuring height on aerial photographs in Lesson 6, and we won't discuss heights any further here. Crown measurements are new for you, so we will spend some time on these.

Crown measurements are useful for predicting tree volume because they are correlated to DBH or basal area. Generally, higher correlations exist for even-aged conifers which have not been subjected to undue suppression or stand competition. The relationship between crown dimension and DBH is usually linear for trees in the middle diameter or age classes. However, crown dimension often does not contribute significantly to the regression equation if tree height is included.

Crown diameters can be measured on aerial photographs in the same manner as any distance. Measurement can be difficult because of the small size of the crown image even on large scale photographs, the effects of crown shadows, and the impact of non-circular crowns. Careful interpreters can measure to within approximately 0.1 mm. The final accuracy of the measurement is dependent upon the scale of the photograph, film resolution, and the ability of the interpreter.

Crown areas can be measured with finely graduated dot grids, using electric planimeters, by assuming various shapes (e.g., oval), or with stereoplotter coordinates along the crown perimeter. At the present time, crown area is more time-consuming to measure than crown diameter and consequently is not used as frequently. However, crown area often is more highly correlated with stem volume than is crown diameter. As more sophisticated electronic equipment becomes available, crown areas may begin to supplant crown diameter in single tree photo volume equations.

Volume tables are often compiled from photo volume equations based upon crown and height measurements. Occasionally, single-entry volume equations based on either crown diameter or crown area are used because of the amount of time involved in determining tree heights. This approach is similar in concept to local volume equations. It works well when tree height is relatively constant within a crown class; this is only to hold likely for the specific area for which the equation was derived.

ESTIMATES OF STAND VOLUME

With medium to small scale aerial photographs, the emphasis is often on measuring stand variables rather than individual tree variables. Stand volume has been developed based on measurements that can be made on aerial photographs. These measurements include species composition, average stand height (height class), average crown diameter, and percentage crown closure. Methods of estimating stand volume, including the use of photo measurements for that purpose are covered in the course Forestry 238.

APPLICATIONS OF PHOTOGRAMMETRY IN TIMBER INVENTORIES

Timber inventories provide the quantity and quality of timber present within a designated area. The purpose of a timber inventory is similar to that of inventories of other commodities (e.g., merchandise within a store). It provides information necessary to the effective management of the commodity. Timber inventories are covered in detail in the course Forestry 238 so we will not go into much detail here.

Timber inventories can cover large geographical areas. Because of the size of the areas, basing the inventory on purely ground-based measurements is normally prohibitively expensive. However, some ground-based measurement may be necessary in order to correct the photo-based measurements.

Aerial photographs are widely used in timber inventories for purposes of stratification (i.e., cover typing) and measurement. Other inventory-related applications include preparation of maps and planning of the sampling scheme. Aerial photographs are also used for updating existing maps to reflect changes to the forest (e.g., fires, cutting, regeneration) since the maps were made.

NON-PHOTOGRAPHIC IMAGING SYSTEMS

As the name implies, **non-photographic imaging systems** are ways of sensing and storing images that do not use film as *both* a sensing device and a storage medium. These types of systems are often called **remote sensing systems.** In the broadest sense, remote sensing refers to techniques used to detect and study objects at a distance without physical contact. Sight, smell, and hearing may all be considered as examples of remote sensing in this context. However, remote sensing is generally used in a more restrictive fashion to describe a series of technological systems used for identifying objects on the ground from an image produced from an aerial or space platform. All remote sensing devices produce an image, even though the images may look considerably different depending upon the device. For example, a camera produces photographs, a thermal scanner produces a thermograph, and certain satellite scanners produce digital codes.

In the material that follows, we will concentrate on providing you with an overview of some non-photographic imaging systems and some of their potential uses in forestry.

Comparison of Photographic and Non-Photographic Imaging Systems

Photographic and non-photographic systems have certain similarities and differences. The best type of system to employ for a specific use depends to a large part on the nature of the use. One way of appreciating how the differences between these two groups of systems affect how they may be best employed is to look at certain general characteristics of the systems.

Photographic imaging systems are limited to a very narrow band of the EMR spectrum extending just beyond the boundaries of visible light, as you learned in Lesson 5. Non-photographic imaging systems can sense in a wide range of bands that include visible light. Bands that are suitable for detecting certain characteristics of interest can be selected. The fact that photographic systems are limited to visible light has mixed effects. On one hand, it is a drawback because photographic systems are not effective if light conditions are poor (e.g., smoke, haze, darkness, etc.). These conditions do not hamper particular types of non-photographic sensors. On the other hand, the photographic process produces images with superior **resolution** than those produced by non-photographic processes. Resolution refers to how well a sensor can record spatial detail. You can think of it as how small an object on the earth's surface can be, and still be seen by a sensor. The better the resolution of the sensor, the smaller the object that can be seen.

Photographic sensors utilize chemical reactions in the emulsion layers of the film to detect, store, and display energy variations within a scene. Non-photographic sensors store electronic signals that correspond to energy variations within a scene. The source of these signals may be either incident (e.g., passive sensors) or self-generated (e.g., active sensors). These signals can be stored on magnetic tape or displayed as an image on a screen and photographed. On some occasions, the images are also electronically enhanced. Note that even though the final images of non-photographic sensors may end up on film, the film is used only for displaying the information.

The fact that non-photographic sensors generate images using electronic signals makes it possible to transmit these images over large distances. For example, satellite sensors detect from space, but the images are transmitted as electronic signals to receiving stations on the earth where they are stored and/or displayed. The electronic nature of the sensed data also makes it amenable to computer analysis. Photographic data are not as easily reduced to machine-readable form.

Computer analysis of images has certain advantages and disadvantages compared to human interpretation of data in the form of visual images. Computer interpretation is rapid and more precise when spectral differences are of primary concern. Also, if several different bands are involved, there are too many variables for the human mind to comprehend simultaneously. However, computer analysis is no better than the computer programs used to make the analysis. Often many factors outside the characteristics of objects themselves affect the signal strength (e.g., topography, season of the year, time of day). These factors must be considered in the interpretive program. Also, spatial patterns are difficult to distinguish using computers; visual interpretation allows the use of the human mind to qualitatively evaluate spatial patterns and make subjective judgements.

Because human interpreters are limited in their abilities to discern spectral patterns and computers are limited in their ability to discern spatial patterns, the best approach to analysis is often a combination of the two. This is called **computer assisted analysis**. In this type of analysis, the computer is provided with proper identification and coordinates of certain ground conditions, and uses this as a guide in analyzing the rest of the image.

EXAMPLES OF NON-PHOTOGRAPHIC IMAGING SYSTEMS

Several non-photographic imaging systems are described below. Neither the list of systems nor the descriptions are exhaustive. It is sufficient for this course for you simply to have a broad understanding of a few of the major types of non-photographic systems. We suggest that you refer to some of the references provided at the end of Chapter 7 in the textbook by Avery and Berlin for further information.

thermal scanner

The **thermal scanner** is a passive type of optical-mechanical scanning system. It is comprised of three components:

- *an optical-mechanical scanner:* this is a mirror mounted on a shaft oriented at a 45 degree angle parallel to the line-of-flight that sweeps the terrain at right angles to the flight path. This component collects radiated energy and focuses it onto the detector.
- *a thermal infrared detector:* different detector elements are used to sense within the various thermal regions of the spectrum. The main atmospheric widows in this range are from 3.5 to 5 μm and between 8 and 14 μm.
- *an image recorder:* this is usually comprised of a magnetic tape and/or a direct film recorder. The recording medium advances at a rate proportional to the aircraft ground speed.

When the thermal scanning image is displayed on standard black-and-white film, the resulting image looks like a conventional photographic image with objects appearing in various shades of grey. The lighter tones represent the warmest radiant temperatures and the darker tones the coolest temperatures. Thermal scanners can be used to locate "hot spots" following fires.

multispectral scanner

The **multispectral scanner** is very similar in design and operation to the thermal scanner. It differs in that it separates incoming radiation into several discrete spectral bands (ranges of wavelengths) that are independently sensed and recorded. The normal storage medium is a multichannel magnetic tape but a cathode ray tube or photographic film may also be used. The advantages of a multispectral scanning system include:

- the entire spectrum range from ultraviolet to the thermal infrared can be sensed with a single optical system at one time;
- data are easier to send electronically from space (no film);
- individual bands can be colour coded and two or more can be combined to produce true colour or an almost unlimited number of false colour renditions of the image (called 'enhancing').

As with all passive sensors, cloud cover can affect the quality of the image. Multispectral scanner images have been used in forestry for land use classification, insect and disease mapping, and other uses that don't require high resolution images.

imaging radar

Radar is an acronym for **ra**dio **d**etection **a**nd **r**anging. It is an active sensing system that supplies radiation in the radio and microwave portion of the electromagnetic spectrum. The common radar system used for interpretation is **side-looking airborne radar** (SLAR). This system creates a two-dimensional image by transmitting and receiving short bursts of energy from the side of a moving aircraft. It operates at wavelengths ranging from 0.8 to 100 cm. The shorter wavelengths produce better resolution, but the longer wavelengths are better for penetration of clouds and haze.

Imaging radar works by electronically measuring the return time of the energy pulses. The range distance between the transmitter and an object can be determined from this. The intensity of the returned pulse is a complex function of the interaction between the terrain and the transmitted pulse. Strongly reflective surfaces appear light in the image if the signal is reflected back directly. Surfaces such as water appear dark because much of the signal is diffused. Advantages of imaging radar include:

- all-weather capacity;
- night sensing capability;
- long lateral coverage (up to 50 km to one side of the aircraft);
- enhancement of geologic features (shadow effect);
- suppression of detail (emphasizes larger terrain features of interest);
- limited geometric distortion.

The major disadvantage of SLAR is its poor resolution. Because of this, it is seldom used at scales larger than 1:125,000. SLAR can be used in forestry for mapping fire boundaries and for preparing inventory maps in regions where haze is common (e.g., tropical rain forests).

satellite imagery

Non-photographic imaging systems can be readily employed from orbiting satellites. The United States began providing satellite-acquired remote sensing data to the public in the late 1960's. LANDSAT (**Land Sat**ellite) represented the first program designed specifically for collecting remote sensing data. (Prior to January 1975, the program was known as ERTS [Earth Resources Technology Satellite].) To date, five LANDSAT missions have been launched — the first in 1972 and the last in 1984. The LANDSAT satellites were launched so as to orbit the earth in repetitive, near-polar, sun-synchronous paths. This results in the satellite returning to the same position after a number of orbits around the planet. All five LANDSATs have carried a multispectral scanner with a ground resolution of about 80 m. The primary sensor of LANDSATs 4 and 5 has been a thematic mapper. The thematic mapper is similar to the multispectral scanner in theory, but it incorporates improved spatial resolution (30 m), additional spectral bands (7), greater radiometric accuracy, and improved geometric fidelity.

Other satellite sensing programs have been established after LANDSAT. Some of these are described in Avery and Berlin (pages 223 to 230). The major limiting factor in any of the satellite-based remote sensing images released to the public has been the ground resolution. The best resolution available is on the order of 10 m from the French SPOT program. Major forestry uses include land use classification and environmental monitoring.

GEOGRAPHIC INFORMATION SYSTEMS

Management of the forest resources requires a large amount of accurate and timely information. This requires the organization and storage of large amounts of data, and the analysis and display of this data in a format that is useful to a large number of users. More and more commonly, a large proportion of the information needs are met by remote sensing data.

Geographic Information System (GIS) is the name given to a computer system designed to accept, organize, store, analyze, and display spatial (i.e., positional) and attribute (i.e., descriptive) information. Both the spatial and the attribute information are digitally referenced to a common coordinate system. Information is stored in different layers called **data bases.** When the data bases are linked together they form a **data bank.**

A number of different GIS packages have been developed over the last several years and are available commercially. In this section five basic elements common to all such systems are discussed.

ENCODING

There are two basic types of position indexing systems used: **raster** (cell) coding and **vector** (polygon) coding. Raster coding can be thought of as a matrix of cells superimposed over a map image. Each location or square is referenced by a digital code. Attribute information is linked to the geographic position through the raster code. Normally, the information category most prevalent within the raster is stored, although more precise procedures are beginning to become available. Raster coding is functionally equivalent to the **pixels** that comprise a remotely sensed digital image. Pixels are squares representing the smallest unit of resolution in an image. The fact that raster coding matches pixel structure facilitates transfer of remotely sensed information to the GIS.

Vector coding consists of digitally encoding and storing points along the perimeter of each distinct area (polygon). This type of coding more accurately defines area boundaries and requires less computer storage space than raster coding. However, vector coding generally must be translated to raster coding if remotely sensed data are to be input directly into the system.

Data Input

Analog data (i.e., maps) need to be converted to digital data (i.e., numbers) by the process of **digitizing** in order to be input into a GIS. Digitizing may be done either manually using a digitizer and a digitizing table or automatically using a vector scanner. Digital data (e.g., satellite images) usually require reformatting and scaling to match the geometry of the GIS image.

Data Management

The data management component of a GIS consists of a series of computer programs that allow for data entry, storage, retrieval and maintenance tasks. This component essentially performs the same functions as data base management computer packages available commercially. In the case of some GIS packages, existing data base management software is incorporated into the system architecture.

Manipulative Operations

One of the major advantages of a GIS over other systems of data storage and retrieval is the ability to quickly manipulate and analyze the data stored within the system. Two kinds of analyses are possible: **surface analysis** and **overlay analysis**. Surface analysis applies to relationships that exist within one data base (level) in the data bank. An example of surface analysis would be determining the area of forest land within different cover types. Overlay analysis applies to relationships that exist among two or more data bases (levels) in the data bank. For example, determining the area of land within a particular cover type that has a particular soil type and range of elevations. (Cover type, soil, and elevation information would all be stored in different data bases related to each other by their geographic positioning.) It is also possible to develop **evaluative models** (used to assess environmental characteristics) and **allocative models** (used to indicate areas best suited for certain uses) through quantifying and weighting the results of overlay analyses.

Output

Data can be displayed in formats that are tabular (e.g., tables) or graphic (e.g., maps, charts). Most systems have facilities for video monitoring and hard copy (printer and plotter) display. Maps highlighting various features of interest can usually be readily produced.

REVIEW/SELF-STUDY QUESTIONS

Do these questions to check your understanding of the material and to review for the final examination.
Do not submit answers to the tutor.

1. In what ways do objects differ from each other in aerial photographs?
2. What is cover typing?
3. What factors make stands appear distinctive and how can they be quantified using aerial photographs?
4. What five features are normally examined stereoscopically to identify tree species on large-scale aerial photographs? Briefly describe what each of these entail.
5. What are photo volume equations? How do they compare with ground-based volume equations?
6. What are the primary causes of the poorer accuracy of photo volume equations?
7. What attributes are commonly measured as independent variables in photo volume equations? Briefly describe how each of these may be measured.
8. What measurements can be made on aerial photographs to help estimate stand volumes?
9. What is a timber inventory?
10. How can aerial photography be used in timber inventories?
11. Define a non-photographic imaging system.
12. What is remote sensing?
13. Compare photographic and non-photographic imaging systems.
14. Compare computer analysis and human interpretation of data in the form of visual images.
15. Briefly describe three examples of non-photographic imaging systems. State examples of the uses of each of these systems in forestry.
16. What is a geographic information system (GIS)?
17. Briefly describe the five basic elements common to all geographic information systems.
18. Differentiate between vector and raster coding. What are the advantages of each?
19. Differentiate between surface analysis and overlay analysis using a geographic information system? Provide an example of each.

APPENDIX A — ASSIGNMENTS TO BE SUBMITTED FOR GRADING

GENERAL INSTRUCTIONS

For all assignments that you submit for grading, be sure to read the assignment carefully. If you are not clear about what is required, discuss the problem with your tutor.

Be sure to submit each assignment so that it reaches your tutor by the due date shown on the course schedule. Include a pink assignment sheet for comments.

Your tutor appreciates receiving assignments that are typed or output on a printer; if you are unable to provide a printed copy, write as legibly as possible using dark-coloured ink.

On rare occasions, assignments are lost in the mail. Keep a copy of each assignment you send in the event of loss.It is also possible that your tutor may wish to discuss an assignment with you over the telephone; it helps if you have a copy you can follow as the tutor refers to it.

GRADED ASSIGNMENT #1 (at end of Lesson 1)

Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date.

This assignment consists of two parts. The first part involves measuring diameter at breast height (dbh) using a diameter tape and measuring tree heights using a Suunto clinometer and a staff hypsometer. This is intended to provide you with some practice using these instruments. The second part consists of some simple statistical analyses that you will perform on the data you collect. This is intended to provide you with an opportunity to review your basic statistics.

PART 1

Prepare a table given the directions below and submit it as part of the assignment.

Select 20 trees on which you can measure dbh and height. Try to select trees from a wide range of sizes. Although it is not essential, it would be best if you select trees from a single softwood species. For each tree selected, record the following information in the order listed:

1. tree number
2. your guess of dbh (record to the nearest centimetre).
3. your measurement of dbh using the diameter tape (record to the nearest 0.1 cm).
4. your guess of tree height (to the nearest metre).
5. estimation of tree height using a staff hypsometer (to the nearest metre).
6. horizontal distance from the tree when using the Suunto clinometer (to the nearest 0.1 m).
7. Suunto clinometer reading to the top of the tree (in either percent or degrees).
8. Suunto clinometer reading to the base of the tree (in either percent or degrees).
9. Height of the tree, based on items 6, 7, and 8 (to the nearest 0.1 m).

PART 2 Instructions 1 through 3 below direct you to build a table of "differences." All the differences should be recorded in the same table. The sign of each of the differences is important; be sure to include the signs in the table.

1. Record in tabular format the differences between your guesses of dbh and measured dbh for each tree. Determine these differences as GUESSED VALUE – MEASURED VALUE. Label this column $Diff_1$.
2. Record in tabular format the differences between your guesses of height and height determined using the Suunto clinometer for each tree. Determine these differences as GUESSED VALUE – SUUNTO VALUE. Label this column $Diff_2$.
3. Record in tabular format the differences between the height determined using a staff hypsometer and the height determined using the Suunto clinometer. Determine these differences as STAFF HYPSOMETER VALUE – SUUNTO VALUE. Label this column $Diff_3$.
4. Determine the mean, standard deviation, and the standard deviation of the mean (standard error) for each column of differences.
5. Determine the percentage deviation of each of the columns by dividing the mean difference by the mean of the measurements using the most accurate instrument (diameter tape for diameter and the Suunto clinometer for height) and multiplying by 100. Which of the columns had the lowest percentage deviation? Provide an explanation for this.
6. Use a paired *t*-test to check whether the mean differences can be said to be different from zero with 95 percent confidence.
7. Prepare graphs of the absolute value of the differences versus tree number. You may include $Diff_2$ and $Diff_3$ on the same graph, but you should show them in different colours if you do. Do your guesses appear to get better as you got some practice?
8. Write a short report (not more than a few pages) presenting your results, and any interpretations that you can draw from the results.

GRADED ASSIGNMENT #2 (at end of Lesson 3)

Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date.

This assignment will help you to practise finding the volume of trees given measurements from a sectioned tree and to fit volume and height functions.

PROBLEM 1

Given the following data for a tree which was felled and sectioned:

Tree Number 1

Section	Section Length (m)	Diameter i.b. (cm)
1	0.30 (stump height)	28.5
2	1.00 (breast height)	26.0
3	1.50	25.9
4	2.50	23.9
5	2.50	21.7
6	2.50	20.8
7	2.50	18.6
8	2.50	15.3
9	2.50	12.5
10	2.50	8.4
11	2.50	4.0
12	1.70	0.0

1. Calculate the volume in m^3 for each tree using (i) Smalian's equation for the middle sections; (ii) the equation for a cone for the top section; (iii) the equation for a cylinder for the base section. Be careful to convert diameter measurements to metres so that areas will be in square metres and show all calculations.

2. Compare the volume obtained using these three equations to the volume that you would calculate if you assumed that the entire tree was cone-shaped.

PROBLEM 2

Using the information given on the next page for ten sample trees, calculate the slope and intercept for each of the following equations:

i) $\text{Volume}_i = \beta_0 + \beta_1 \times \text{dbh}^2_i \times \text{height}_i + \varepsilon_i$

ii) $\text{Height}_i = \beta_0 + \beta_1 \times \text{dbh}^2_i + \varepsilon_i$

Hint: Because dbh^2 times height results in a large number and volume is a small number, you can make the calculations more accurate and easier to do if you relate volume to dbh^2 times height divided by 1000.

Explain why or why not you think each of the six assumptions of simple linear regression have been met for each equation.

For the height estimation equation only:

i) Calculate a 95% confidence interval for the true intercept and the true slope.

ii) Test whether the true intercept could be 1.3 m (e.g., the height equals breast height when dbh is zero).

iii) Calculate the r^2 value and the standard error of the estimate. Explain what each of these terms means for the height regression.

iv) Use two different methods to test whether the regression is significant. Draw a conclusion.

Tree Data

dbh (cm)	Height (m)	Volume (m^3)
13.4	13.90	0.0939
16.8	14.70	0.1604
9.7	8.80	0.0308
27.0	16.80	0.3575
19.0	16.54	0.2486
37.3	28.00	1.3623
23.6	18.21	0.3556
28.1	15.70	0.4928
13.1	13.41	0.0873
39.5	25.70	1.3664

PROBLEM 3 Using the fitted volume equation from Problem 2, estimate the volume of the first tree given in Problem 1. How does this estimated volume compare to the volume you calculated for Problem 1? Explain any differences.

GRADED ASSIGNMENT #3 (at end of Lesson 4)

Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date. The assignment consists of two problems.

PROBLEM 1

This exercise is designed to test your understanding of the process of determining log volume and reducing log dimensions for rot. You will probably need to use the examples provided in the lesson to get started. See if you can complete some of the later logs without having to look at your notes.

For each of the following logs, determine:

1. the exact firmwood volume;
2. the reduced dimensions a scaler would record using radii reduction and the associated volume (only if it is appropriate to use this procedure);
3. the reduced dimensions a scaler would record using length reduction and the associated volume (only if it is appropriate to use this procedure).

It is to your advantage to show your calculations. If you make a mistake, the tutor should be able to show you where you went wrong. Explanations of what the vectors represent can be found in your course notes.

Log #	Log Vector	Rot Vector	Rot Type
1	9.6/45/57	9.6/ 4/12	Heart Rot
2	13.4/35/49	5.0/ / 8	Heart Rot
3	8.2/24/35	8.2/ 4/ 6	Sap Rot
4	10.4/15/28	10.4/ 5/10	Heart Rot
5	9.8/45/65	/ /20	Butt Rot
6	7.8/34/47	7.8/22/33 7.8/10/15	Ring Rot
7	6.6/22/30	3.0/ / 6	Heart Rot
8	14.8/15/32	/ / 8	Sap Rot
9	10.4/40/52	10.4/30/40 10.4/24/33	Ring Rot
10	7.2/21/33	7.2/ 2/ 4	Heart Rot

PROBLEM 2

Many logs sold outside British Columbia are sold in terms of their estimated board foot content. Board foot content varies depending upon the log rule employed, and may also be affected by the length of the log. In this problem, logs are for export and the buyer and seller have agreed upon Scribner Decimal C, the most common rule in the Pacific Northwest, as an acceptable log rule. However, the Scribner Decimal C log rule is affected by log length since it employs only diameter at the top end of the log. Any taper between the bottom and top of the log is assumed to be waste. Because of this assumption, a long single log is scaled with fewer board feet than the same log bucked into a number of smaller logs. This problem allows you to explore this issue and provides some exposure to a common log rule.

You intend to sell a load of logs to a potential buyer who has agreed to pay you \$250/1000 bf if the scaling is done with the Scribner Decimal C log rule. Since you have the opportunity to buck your logs into shorter pieces before the scaling, you intend to make the most of it.

A representative sample yielded the following three logs which you will use to determine your most profitable bucking strategy.

Log #	Top Diameter (in.)	Length (feet)	Bottom Diameter (in.)
1	30	24	35
2	40	28	46
3	50	32	60

Determine for each log:

1. solid content in m^3;
2. board foot scale without further bucking;
3. best bucking strategy, board foot scale of the sum of all pieces, and the percentage gain for each log;
4. value of the log with and without further bucking;
5. number of board feet per m^3 with and without further bucking;
6. value per m^3 with and without further bucking.

Describe in a few sentences the general bucking strategy that you followed for the three logs.

hints

You should use Smalian's formula for determining the solid wood content in m^3. Tabulated values of Scribner Decimal C log rule follow on pages 159 and 160. These values are in 10's of board feet. The diameter in the tables is the top end diameter of the log. You can reasonably assume that logs resemble conical sections to get intermediate diameters when bucking. As an example consider a log with a top diameter of 30 inches, a bottom diameter of 40 inches, and a length of 20 feet. A conical section implies that the taper is constant. The taper for this log is $(40 - 30) \div 20$ which equals 0.5 inches per foot. The diameter 8 feet from the bottom of the log can be calculated to be $40 - 0.5 \times 8$ which equals 36 inches. All diameters need to be rounded to the nearest inch. If the diameter is exactly half way between the two nearest inches, then round to the even-numbered inch. For example, 25.5 inches would be rounded to 26 inches; 26.5 inches would also be rounded to 26 inches. No logs can be less than 8 feet long.

SCRIBNER DECIMAL C LOG RULE

Source: J.R. Dilworth. 1973. *Log Scaling and Timber Cruising.* Corvallis OR: Oregon State University Book Store.(pages 386–389)

TABLE 1.—*Scribner Decimal C log rule* 1/

6- TO 16-FOOT LOGS / 17- TO 32-FOOT LOGS

Diameter, inches	Length—feet									Length—feet								
	8	9	10	11	12	13	14	15	16	17	18	20	22	24	26	28	30	32
	Contents—board feet in tens									Contents—board feet in tens								
6	0.5	0.5	1	1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
7	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	5	5	6
8	1	1	2	2	2	2	2	2	3	3	3	3	4	4	5	6	6	7
9	2	2	3	3	3	3	3	3	4	4	4	4	5	6	6	7	8	9
10	3	3	3	3	3	4	4	5	6	6	6	7	8	9	9	10	11	12
11	3	3	4	4	4	5	5	6	7	7	8	8	9	10	11	12	13	14
12	4	4	5	5	6	6	7	7	8	8	9	10	11	12	13	14	15	16
13	5	5	6	7	7	8	8	9	10	10	11	12	13	15	16	17	18	19
14	6	6	7	8	9	10	10	11	11	12	13	14	16	17	19	20	21	23
15	7	8	9	10	11	12	12	13	14	15	16	18	20	21	23	25	27	28
16	8	9	10	11	12	13	14	15	16	17	18	20	22	24	26	28	30	32
17	9	10	12	13	14	15	16	17	18	20	21	23	25	28	30	32	35	37
18	11	12	13	15	16	17	19	20	21	23	24	27	29	32	35	37	40	43
19	12	13	15	16	18	19	21	22	24	25	27	30	33	36	39	42	45	48
20	14	16	17	19	21	23	24	26	28	30	31	35	38	42	45	49	52	56
21	15	17	19	21	23	25	27	28	30	32	34	38	42	46	49	53	57	61
22	17	19	21	23	25	27	29	31	33	35	38	42	46	50	54	58	63	67
23	19	21	23	26	28	31	33	35	38	40	42	47	52	57	61	66	71	76
24	21	23	25	28	30	33	35	38	40	43	46	50	55	61	66	71	76	81
25	23	26	29	31	34	37	40	43	46	49	52	57	63	69	75	80	86	92
26	25	28	31	34	37	41	44	47	50	53	56	62	69	75	82	88	94	100
27	27	31	34	38	41	44	48	51	55	58	62	68	75	82	89	96	103	110
28	29	33	36	40	44	47	51	54	58	62	65	73	80	87	95	102	109	116
29	31	35	38	42	46	49	53	57	61	65	68	76	84	91	99	107	114	122
30	33	37	41	45	49	53	57	62	66	70	74	82	90	99	107	115	123	131
31	36	40	44	49	53	58	63	67	71	75	80	89	98	108	115	124	133	142
32	37	41	46	51	55	60	64	69	74	78	83	92	101	110	120	129	138	147
33	39	44	49	54	59	64	69	73	78	83	88	98	108	118	127	137	147	157
34	40	45	50	55	60	65	70	75	80	85	90	100	110	120	130	140	150	160
35	44	49	55	60	66	71	77	82	88	93	98	109	120	131	142	153	164	175
36	46	52	58	63	69	75	81	86	92	98	104	115	127	138	150	161	173	185
37	51	58	64	71	77	84	90	96	103	109	116	129	142	154	167	180	193	206
38	54	60	67	73	80	87	93	100	107	113	120	133	147	160	174	187	200	214
39	56	63	70	77	84	91	98	105	113	119	126	140	154	168	182	196	210	224
40	60	68	75	83	90	98	105	113	120	128	135	150	166	181	196	211	226	241
41	64	72	79	87	95	103	111	119	127	135	143	160	175	191	207	223	238	254
42	67	76	84	92	101	109	117	126	134	143	151	168	185	201	218	235	252	269
43	70	79	87	96	105	113	122	131	140	148	157	174	192	209	227	244	262	279
44	74	83	93	102	111	120	129	139	148	157	166	185	204	222	241	259	278	296
45	76	85	95	104	114	123	133	143	152	161	171	190	209	228	247	266	285	304
46	79	89	99	109	119	129	139	149	159	169	178	198	218	238	258	278	297	317
47	83	93	104	114	124	134	145	155	166	176	186	207	228	248	269	290	310	331
48	86	97	108	119	130	140	151	162	173	184	194	216	238	260	281	302	324	345
49	90	101	112	124	135	146	157	168	180	191	202	225	247	270	292	314	337	359
50	94	105	117	129	140	152	164	175	187	199	211	234	257	281	304	329	351	374

1/ U.S. Forest Service. See Table 6 for Columbia River Scaling Bureau Log Rule Table.

SCRIBNER DECIMAL C LOG RULE

continued

	8- TO 16-FOOT LOGS									17- TO 32-FOOT LOGS								
Diameter, inches	Length—feet									Length—feet								
	8	9	10	11	12	13	14	15	16	17	18	20	22	24	26	28	30	32
	Contents—board feet in tens									Contents—board feet in tens								
51	97	110	122	134	146	158	170	183	195	207	219	243	268	292	316	341	365	389
52	101	114	127	139	152	165	177	190	202	215	228	253	278	304	329	354	380	405
53	105	118	132	145	158	171	184	197	210	224	237	263	289	316	341	368	395	421
54	109	123	137	150	164	177	191	205	218	232	246	273	300	328	355	382	410	437
55	113	127	142	156	170	184	198	212	227	241	255	283	312	340	368	397	425	453
56	118	132	147	162	176	191	206	220	235	250	264	294	323	353	382	411	441	470
57	122	137	152	167	183	198	213	228	244	259	274	304	335	365	396	426	457	487
58	126	142	158	174	189	205	221	237	253	268	284	316	347	379	410	442	473	505
59	131	147	163	180	196	212	229	245	261	278	294	327	359	392	425	457	490	523
60	135	152	169	186	203	220	237	253	270	287	304	338	372	406	439	473	507	541
61	140	158	175	193	210	228	245	263	280	298	315	350	385	420	455	490	525	560
62	145	163	181	199	217	235	253	271	289	307	325	362	398	434	470	506	542	579
63	149	168	187	205	224	243	261	280	299	317	336	373	411	448	485	523	560	597
64	154	174	193	213	232	251	270	290	309	329	348	387	425	464	503	541	580	619
65	159	179	199	219	239	259	279	299	319	339	358	398	438	478	518	558	597	637
66	164	185	206	226	247	268	288	309	329	350	370	412	453	494	535	576	617	659
67	170	191	212	233	254	275	297	318	339	360	381	423	466	508	550	593	635	677
68	175	197	219	240	262	284	306	328	350	371	393	437	480	524	568	611	655	699
69	180	203	226	248	271	294	316	339	361	384	406	452	497	542	587	632	677	723
70	186	209	232	256	279	302	325	349	372	395	419	465	512	558	605	651	698	744
71	192	215	240	263	287	311	335	359	383	407	430	478	526	574	622	670	717	765
72	197	222	247	271	296	321	345	370	395	419	444	493	543	592	641	691	740	789
73	203	229	254	280	305	330	356	381	406	432	457	508	559	610	661	712	762	813
74	209	236	261	288	314	340	366	393	418	445	471	523	576	628	680	733	785	837
75	215	242	269	296	323	350	377	404	430	458	484	538	592	646	700	754	807	861
76	221	249	277	304	332	360	387	415	443	470	498	553	609	664	719	775	830	885
77	228	256	285	313	341	369	398	426	455	483	511	568	625	682	739	796	852	909
78	234	263	293	322	351	380	410	439	468	497	527	585	644	702	761	819	878	936
79	240	271	301	331	361	391	421	451	481	511	541	602	662	722	782	842	902	963
80	247	278	309	340	371	402	432	463	494	525	556	618	680	742	804	866	927	989
81	254	286	317	349	381	413	444	476	508	540	572	635	699	762	826	889	953	1,016
82	261	293	326	358	391	424	456	489	521	554	586	652	717	782	847	912	977	1,043
83	268	301	335	368	401	434	468	501	535	568	601	668	735	802	869	936	1,002	1,069
84	275	309	343	378	412	446	481	515	549	584	618	687	755	824	893	961	1,030	1,099
85	281	316	351	386	421	456	491	526	561	596	631	702	772	842	912	982	1,052	1,123
86	287	323	359	395	431	467	503	539	575	611	646	718	790	862	934	1,006	1,077	1,149
87	295	332	368	405	442	479	516	553	589	626	663	737	810	884	958	1,031	1,105	1,179
88	301	339	377	414	452	490	527	565	603	640	678	753	829	904	979	1,055	1,130	1,205
89	308	347	385	424	462	501	539	578	616	655	693	770	847	924	1,001	1,078	1,155	1,232
90	315	354	393	433	472	511	551	590	629	669	708	787	865	944	1,023	1,101	1,180	1,259
91	322	362	402	443	483	523	563	604	644	684	725	805	886	966	1,047	1,127	1,208	1,288
92	329	370	411	453	493	534	575	616	657	698	740	822	904	986	1,068	1,150	1,233	1,316
93	335	377	419	461	503	545	587	629	671	713	755	839	922	1,006	1,090	1,174	1,258	1,341
94	343	386	428	471	514	557	600	643	686	728	771	857	942	1,028	1,114	1,199	1,285	1,371
95	350	394	437	481	525	569	612	656	700	744	788	875	963	1,050	1,138	1,225	1,313	1,400
96	357	402	446	491	536	581	625	670	715	759	804	893	983	1,072	1,161	1,251	1,340	1,429
97	364	410	455	501	546	592	637	683	728	774	819	910	1,001	1,092	1,183	1,274	1,365	1,456
98	371	418	464	511	557	603	650	696	743	789	836	928	1,021	1,114	1,207	1,300	1,392	1,485
99	379	426	473	521	568	615	663	710	757	805	852	947	1,041	1,136	1,231	1,325	1,420	1,515
100	386	434	482	531	579	627	675	724	772	820	869	965	1,062	1,158	1,255	1,351	1,448	1,544

GRADED ASSIGNMENT #4 (at end of Lesson 5)

Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date.

For this assignment, you will require the set of aerial photographs BC 79046 #s 36 – 39 (available from the UBC Bookstore), a UBC campus street map, and portions of the UBC campus survey map. The street map and two photocopied sheets of the survey map are included with your course materials.

This assignment is comprised of two parts. You do not need to submit any material from the first portion. However, you should do this work carefully at this time as it is necessary for the assignments that follow. It is also pertinent to the material we presented in the commentary. If you don't understand it after trying the assignment, you should contact your tutor to clear up any problems you may be having before proceeding to the next lesson.

The accuracy with which you prepare your photographs will affect the accuracy of measurements you will be making later. This is particularly true for locating the principal points and conjugate principal points. Take your time and do it carefully.

PART 1

1. Through your stereoscope, look at the stereogram test patterns provided on page163. Be sure to set the stereoscope width to match your IPD. If you can make out any patterns, then you can see stereoscopically. Take a break if you can't make out any patterns after a few minutes. If you still can't see any patterns after several tries, contact your tutor.
2. Try to view an adjacent pair of your aerial photographs stereoscopically. We suggest reading the appropriate section in the commentary again before you do this. You may have more difficulty seeing stereoscopically on your photographs than you did on the stereograms, but it is only a matter of your eyes getting accustomed. It will become much easier for you with only a little bit of practice. If you are not able to view your photographs stereoscopically after several tries, contact your tutor.

The next several directions pertain to preparing your photographs. Descriptions of what each of these steps entails is included in the commentary for this lesson.

3. Trim the frames from your photographs.
4. Locate and mark the principal point on each photograph.
5. Locate and mark the conjugate principal points on each photograph. Note that you will be able to locate only one conjugate principal point on photos 36 and 39 because these photographs represent the ends of the flight line as far as the assignment is concerned.
6. Locate and mark the line-of-flight on each photograph.
7. Measure and mark the photo base distances and average photo base distances on the rear of the appropriate photographs.

PART 2

1. Compute an average scale (RF) for photo 38 from ratios of photo and ground distance. Use the portion of the survey map included with your course materials to provide accurate ground distances. Note that the survey map is not drawn to an exact scale, but provides point-to-point distances as marked. Work with combinations of these distances to calculate the ground distance corresponding to distances that you can recognize on the photograph. The map provides ground distances in feet. You should convert these distances to metres prior to comparing them to distance measurements on the photograph which you should make in millimetres. A street map of the UBC campus, also included with your course materials, can be used to help you identify specific roads. Compute the RF scale for 10 pairs of distances. The average scale (RF) can be determined by comparing the total ground distance to the total photo distance.

To assist you in recording your results, complete a table with the following headings:

DESCRIPTION OF LINE	GROUND DISTANCE (m)	PHOTO DISTANCE (mm)	RF

2. Will the answer you obtained above be different than the answer you would obtain by averaging the individual scale calculations for each line? Why?

3. If you completed the same table for photo 39, would you expect to get the same RF for lines that appear in both photographs? Explain.

4. Convert the average RF scale you determined in question 1 to the following units:

cm/m	cm on photo per m on the ground
cm/km	cm on photo per km on the ground
cm^2/ha	square cm on photo per hectare on the ground (There are 10,000 m^2 in 1 hectare.)

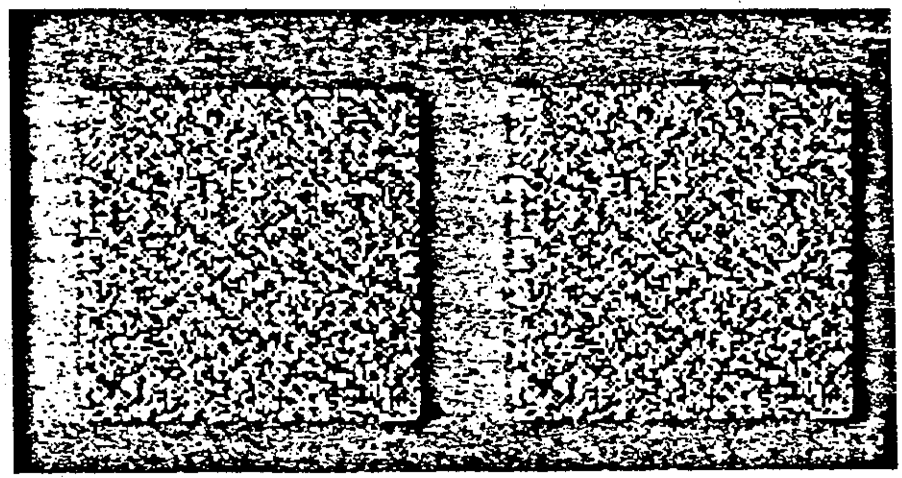
5. The camera focal length is known to be 305 mm. Use the average scale determined in question 1 to estimate the average flying height of the aircraft above the ground.

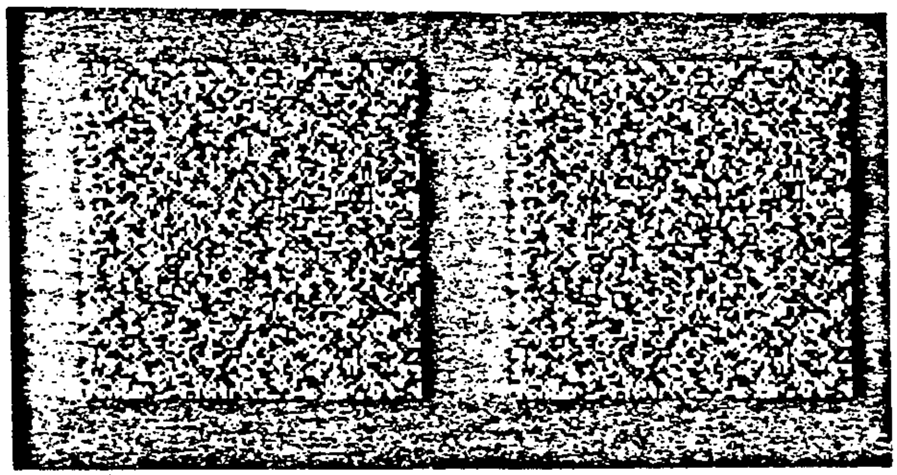
6. Use the results of question 5 to calculate an approximate flying height above mean sea level. (Note that the elevation in feet above mean sea level for selected points can be obtained from the survey map used in question 1. Average the elevation of several points and convert this to metres to get an approximate ground elevation.)

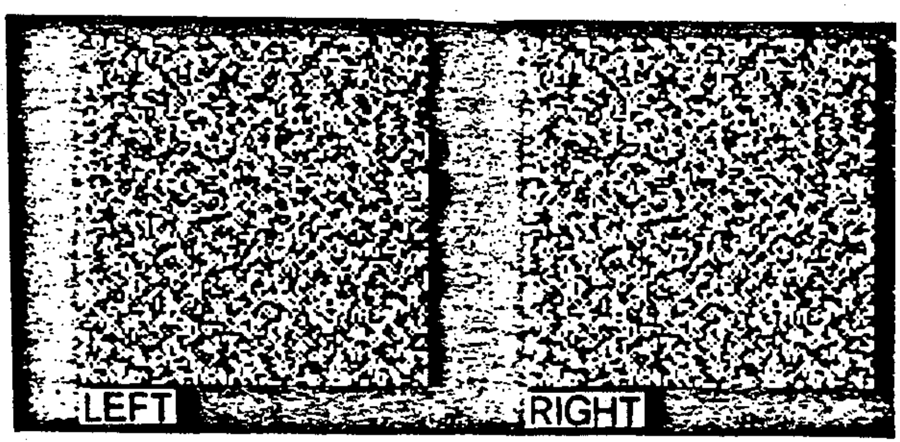
7. Assume that the distance between the principal point on Photo 1 and the conjugate principal point corresponding to the principal point on Photo 2 is larger than the distance between the principal point on Photo 2 and the conjugate principal point corresponding to the principal point on Photo 1. Which principal point has the higher elevation? Explain your reasoning. (A diagram might make your explanation easier.)

STEREOGRAM TEST PATTERNS

for use with Graded Assignment 4, part 1







GRADED ASSIGNMENT #5 (at end of Lesson 6)

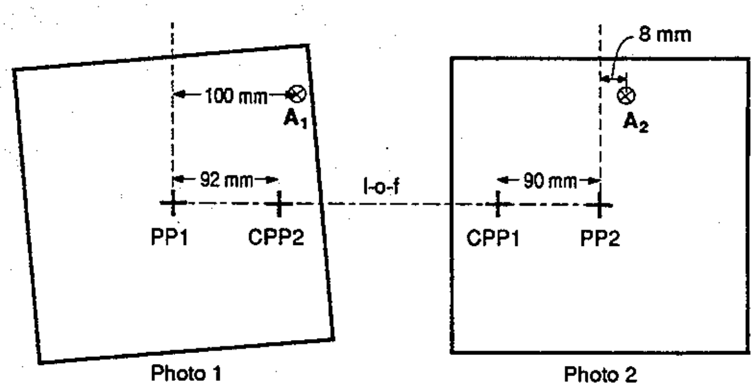
Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date.

There are two parts to this assignment. The questions in Part I are designed to test your understanding of the theory covered in Lesson 6. The problem in Part II will give you an opportunity to practice using the floating mark stereometer.

PART 1

1. This question requires you to consider the subjective "sense-of-depth" achieved in stereoscopic viewing of vertical aerial photography. For each proposed change, indicate whether the sense of depth is likely to: (a) increase; (b) decrease; (c) remain unchanged; or (d) change, but in an unknown way. Briefly justify each answer.
   i) The flying height is increased.
   ii) The photograph is taken at noon with the sun directly overhead (i.e., no shadows).
   iii) The ground distance between successive exposures is increased.
   iv) The photographs are of forest land, but it is a windy day and the tree tops are swaying.
   v) The focal length of the lens is increased.

2. Two overlapping vertical aerial photographs have been prepared for stereo viewing and taped in place as shown in the figure below.



Diagrams of two photographs for use in question 2.

The distance between the principal point of Photo 1 (PP1) and the conjugate principal point of Photo 2 (CPP2) is 92 mm. The distance between the principal point of Photo 2 and the conjugate principal point of Photo 1 (CPP1) is 90 mm. A ground control marker has been located stereoscopically at A1 in Photo 1 and A2 in Photo 2. The distance between A1 and PP1 is 100 mm, measured parallel to the line-of-flight. The distance between A2 and PP2 is 8 mm, also measured parallel to the line-of-flight.
   i) Which principal point has the higher elevation? How do you know this?
   ii) Determine the absolute stereoscopic parallax at the ground control marker.
   iii) A radar altimeter has determined that the flying height above PP1 is 1000 m. As part of your ground survey, you have determined that the precise elevation at the ground control marker is 600 m above mean sea level (MSL). What is the precise elevation above MSL at PP1?
   iv) What is the precise elevation above MSL at PP2?

PART 2 Determine the height of five objects on campus (e.g., buildings, trees, etc.) using the floating mark stereometer provided in your equipment kit. Measure each parallax difference twice and calculate a separate estimate of height based on each of these measurements. Try to ensure that your measurements are independent of one another. You will not be graded on the closeness of your repeated measurements to one another, although extremely large differences (e.g., > 10 m) probably indicate that you made a mistake in either your measurements or your calculations.

Complete a table to contain your 10 height estimates (i.e., five objects each measured twice). Suggested headings are given below. A map of the UBC campus is included with this assignment to help you to describe the location and/or identify the objects whose heights you are estimating.

STEREO PAIR	FLYING HEIGHT (m)	PHOTO BASE (mm	DESCRIPTION OF OBJECT	dP (mm)	HEIGHT OF OBJECT (m)

Assume that the flying height above the base of each object is 2930 m. Use the average photo base distances calculated in the last assignment to approximate absolute stereoscopic parallax at the base of each object.

GRADED ASSIGNMENT #6 (at end of Lesson 7)

Please submit your answer for this assignment to the tutor for marking. Check the course schedule for the due date.

You will be making a partial map of the UBC campus in this assignment. This will provide you with an opportunity to practice transfer of details from aerial photographs to a map using radial line triangulation.

There should be a base map and four mylar sheets in your laboratory kit. Three ground control points (GCP's) have been marked on the base map. Their locations have been marked on the street map of the UBC campus.

1. Locate and mark each GCP on all the photographs on which they are found.

2. Select, locate, and mark suitable photo control points (PCP's) on all the photographs.

3. Use radial line triangulation (described in Lesson 7) to transfer the locations of each of the principal points from the photographs to the base map. Be sure to label each of these.

4. Use radial line triangulation to locate the following features on the map:
   a. clock tower (single point)

   b. Student Union Building (simple rectangle)

   c. Acute Care Hospital (complex shape)

   d. University Blvd. from its junction with East Mall at GCP B out to the edge of stereoscopic coverage at the right edge of photo 37 (continuous line)

Please submit your map, photographs, and mylar sheets. These will be returned to you during the weekend laboratory session.

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